DYNAMIC SIMULATION OF A METAL-BELT CVT UNDER TRANSIENT CONDITIONS

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ABSTRACT

Most of the technical literature on CVT consider steady-state conditions. Very few analyze the behavior of CVT under transient conditions (Kanehara et al., 1999; Ide et al., 1995; Carbone et al., 2002) and none of them include the effects of pulley deformation. This paper is an attempt to fill the gap. It is herein presented a mathematical model of a V-belt metal CVT, valid also under transient conditions, where the pulley deformation has been included. The model, implemented in a numerical code, combines Gerbert’s theory of V belts and the contributions of (Sattler, 1999a; Sattler, 1999b) on pulley deformation. An innovative algorithm for the simulation of dynamic effects during shifting is proposed. Inertia effects as well as deformation of pulleys are taken into account. Two type of working conditions have been experimentally monitored and numerically simulated. Finally, the paper contains a description of the experimental apparatus used for the validation of the model herein presented.

1 Introduction

During last years the technology concerning continuously variable transmission (C.V.T.) has been improved by the synergic combination of mechanical reliability improvements and new strategies of the electronic control systems. Presently CVTs are widely used in automotive solutions and the trend seems increasing. Moreover, CVTs may represent alternative solutions to traditional gearbox to make car driving more comfortable and increase fuel efficiency. Most of the dynamical modeling of CVT, concentrate on steady state conditions or kinematic problems. Very few are the contributions (Carbone et al., 2002) which consider transient conditions. It is obvious that the monitoring of the mechanical efficiency under transient behaviour is important to improve performance especially in automotive applications. The knowledge of the CVT behavior under different working cases is a requirement for the improvement of the control unit in order to attain efficiency and performances. For instance, in the Van Doorne type CVT, by controlling the axial thrust and position of the half-pulleys one can regulate the transmission and torque ratio. For this purpose, the authors extended the models presented by (Gerbert, 1972; Gerbert,Sorge, 1996) and (Sattler, 1999a; Sattler, 1999b) for the analysis of such a type of transmission. Based on this new model, a code has been developed and the numerical results compared with those experimentally obtained.

2 Nomenclature

- \( C_1 \): driving torque;
- \( C_2 \): torque exerted by the belt on the 2nd pulley;
- \( C_3 \): resisting torque;

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3 Transmission mechanics

In a generic metal V-belt CVT the torque is transmitted by oil-lubricated contact between pulley groove and the lateral face of belt elements. Low friction in oil-lubricated CVT requires high axial forces. This leads to significant deformation of shaft and pulleys. In the present model we assume the belt to be a continuous. This hypothesis leads to acceptable results regarding the amount of transmitted torque. However, since the model does not split global tension over the CVT components, the computed belt tensions are not reliable.

Coulomb friction law with constant coefficient $\mu$ and the persistence of a sliding arc along all winding arc are also assumed.

As already said, in a metal belt CVT, torque is transmitted by friction between lateral faces of belt and pulley groove. Thus the torque is transmitted by means of the force component having the same direction of the relative sliding velocity between pulley and belt. Sliding is caused by varying tensile forces and contact forces in winding arcs. Also change of nominal radius (see figure 4) and pulley deformation contribute to sliding motion.

3.1 Belt equations

The following treatment has been inspired by the paper of (Sattler, 1999a). However, the terms required for an analysis during transient conditions are missing in the cited reference, but herein included.

With reference to the scheme presented in figure 1, the effective sliding angle $\beta_s$ can be expressed by means of the following equation:

$$\tan \beta_s = -\tan \beta \cdot \cos \gamma .$$

The equations of motion are deduced by imposing the equilibrium, along the pulley radial and tangent directions, of all the forces acting on a belt element.

In particular, with reference to the scheme shown in figure 1, the acting forces can be classified as follows: belt tension, friction forces, inertia forces and contact reactions. Applying the equilibrium condition along the tangent direction (see figure 2) we obtain:

$$(F + dF) \cos \frac{d\phi}{2} - F \cos \frac{d\phi}{2} + 2 \mu dN \cos \beta_s \sin (\pi + \gamma) - q \dot{\omega} r^2 d\phi - 2 \omega \frac{dr}{dt} r d\phi = 0 .$$
Solving the equation (3) for \( \frac{dN}{d\phi} \) and replacing in (2), we obtain:

\[
\frac{dN}{d\phi} = \frac{F - q r^2 \omega_x + q r \frac{dr}{dt} \omega_y}{2 [\sin \beta + \mu \cos \beta_x \cos (\gamma + \pi)]}
\]  

\[ \frac{dF}{d\phi} = - \left( F - q r^2 \omega_x + q r \frac{dr}{dt} \omega_y \right) \frac{\mu \cos \beta_x \sin (\gamma + \pi)}{\sin \beta + \mu \cos \beta_x \cos (\gamma + \pi)} + q \omega_x r + 2 q \omega_y \frac{dr}{dt}
\]

The axial thrust on the halves of pulley can be computed solving the equation:

\[
\frac{dF_{ax}}{d\phi} = \left( F - q r^2 \omega_x^2 + q r \frac{dr}{dt} \omega_y \right) \frac{\cos \beta - \mu \sin \beta_x}{2 [\sin \beta + \mu \cos \beta_x \cos (\gamma + \pi)]}
\]

To solve equation (5) and (6) \( \beta_x \) or \( \gamma \) is needed. Since they both appear in equation (1) only one of them is required. As shown in the right side of figure 2, the belt velocity \( v \) follows from the vectorial sum of the pulley speed at the contact point \( r\omega \) and the sliding velocity \( v_y \).

The sliding angle \( \gamma \) can be determined using the following expression

\[
\tan \gamma = \frac{v_{y,\text{tan}}}{v_{y,\text{rad}}}
\]

where

\[
v_{y,\text{tan}} = v \cos \theta - r \omega \sim v - r \omega + r \psi
\]

with

\[
v = \left( 1 + \frac{F}{EA} \right) V.
\]

To compute \( v_{y,\text{rad}} \) we can use

\[
\frac{dr}{dt} = \frac{\partial r}{\partial t} + \frac{\partial r}{\partial \phi} \frac{d\phi}{dt} = v_{y,\text{rad}}.
\]

\[\text{An expression for } r \text{ will be provided in the next section (see eq. (21)).}\]
Thus, from (7), the following equation can be deduced (Carbone et al., 2002)

$$\tan \gamma = \frac{(1 + \frac{F}{EA}) V - r \omega + r\psi}{\frac{dr}{dt} + \frac{dr}{d\theta} \omega}, \quad (11)$$

Figure 4. Sliding due to the change of nominal radius

In the actual computations the term $r\psi$ has been computed neglecting the elastic deformation of the belt, but taking into account the sliding to the change of nominal radius, as shown in Figure 4. Although the value of $\theta$ is not constant, we neglect its variations and we assume for it small values, as in equation (8). At each time step, once computed, $V$ is assumed constant along all the winding arcs. In particular, chosen a reference point $R_0$, (11) can be rewritten in the form

$$\tan \gamma = \frac{\frac{EA + F}{EA} \left[ \tan \gamma_0 \left( \frac{dr}{dt} + \frac{dr}{d\theta} \omega \right) + r_0 \omega \right] - r \omega + r\psi}{\frac{dr}{dt} + \frac{dr}{d\theta} \omega}, \quad (12)$$

where the variables with the subscript 0 refer to $R_0$.

According to (Sferra, 2001), when $R_0$ denotes the middle point between the pulleys centers, the following expression can be obtained:

$$\tan \gamma = \frac{\left( \frac{EA + F}{EA} \right) \left( \tan \gamma_0 \left|_{R_0} \right. \right) + r_0 \omega}{\left( \frac{EA + F}{EA} \right) \left( \tan \gamma_0 \sin \theta_0 + \cos \theta_0 \right) - r \omega + r\psi}, \quad (13)$$

where $\tan \gamma_0$ and $\frac{dr}{dt} \big|_{R_0}$ can be computed by means of the following expressions

$$\tan \gamma_0 = \frac{(1 + \frac{F_0}{EA}) R_1 \omega_1 \cos \theta_0 - r_0 \omega_1}{\frac{dr}{dt} \big|_{R_0} - \left( 1 + \frac{F_0}{EA} \right) V \sin \theta_0} \quad ; \quad \quad (14)$$

with $i = 1, 2$ (see figures 5 and 6 for the convention used for the measurement of angle $\theta$), and

$$\frac{dr}{dt} \bigg|_{R_0} = \frac{h}{2} \left( \frac{R_1 + R_2}{h^2 - (R_1 - R_2)^2} \right) \left[ \left( \frac{dr}{dt} + \frac{dr}{d\theta} \right) \sqrt{1 + \left( \frac{dr}{dt} + \frac{dr}{d\theta} \right)^2} - \frac{h (R_1 + R_2)^2}{[h^2 - (R_1 - R_2)^2]^2} \right] - v_0 \sin \theta_0, \quad (15)$$

where

$$\begin{align*}
R_1 &= \tau R_2, \quad (16a) \\
\frac{dR_1}{dt} &= R_2 \frac{d\tau}{dt} + \tau \frac{dR_2}{dt}, \quad (16b) \\
\frac{d\tau}{dt} &= \frac{\tau (t) - \tau (t - \Delta t)}{\Delta t}. \quad (16c)
\end{align*}$$

Figure 5. How to measure angle $\theta$ in the case of the driver pulley ($i = 1$)

Figure 6. How to measure angle $\theta$ in the case of the driven pulley ($i = 2$)

The figures 5 and 6 show how to measure the angle $\theta$ when dealing, respectively, with the primary ($i = 1$) and the secondary ($i = 2$) pulleys.

The details regarding the above derivations can be found in (Sferra, 2001).
3.2 Pulley deformation

To take into account the deformation of pulleys caused by the force between belt faces and pulley’s groove some expressions have been empirically obtained from the results of several run of F.E.M. models of a CVT.

\[ \delta = \begin{cases} 0.00045C_1 & \text{for driving pulley} \\ \frac{0.00045}{\tau^{0.55}}C_1 & \text{for driven pulley} \end{cases} \]  

(20a) 

(20b)

for \( \delta \) expressed in radians and \( C_1 \) in N-m.

Finally, adding the deformations due to shaft deflection and \( w \), as explained in the geometry of figure 7, the local radius \( r \) is given by

\[ r = R \frac{\tan \beta_c}{\tan \beta} \frac{w}{2 \tan \beta} . \]  

(21)

4 Algorithm

The Fortran code developed requires the following data:

- Distance \( b \) between pulleys
- Undeformed belt length \( l_b \)
- Axial stiffness of belt \((AE)\)
- Belt mass density \( q \) for unit of length
- Angle of pulley groove \( \beta_c \)
- Input torque \( C_1 \) and rotational velocity \( \omega_i \)
- Target speed ratio \( \tau \)

The computations can be divided in the following main steps executed at each time interval.

1. Compute \( R_1 \) and \( R_2 \) solving the system

\[ \pi R_2 (\tau + 1) + 2R_2 (\tau - 1) \sin^{-1} \left( \frac{R_2 \tau - 1}{h} \right) \]

\[ + 2 \sqrt{h^2 - R_2^2 (\tau - 1)^2} = l_b , \]  

(22a) 

(22b)

2. Compute \( F_a \), \( F_b \) and \( \alpha_{i,oc} \) with the appropriate set of equations.
- Case $\tau < 1$

\[
F_a = q\omega_1^2 R_1 + (F_b - q\omega_1^2 R_1) \ e^{\omega_1 t} \\
F_a = q\omega_2^2 R_2 + (F_b - q\omega_2^2 R_2) \ e^{\omega_2 (t - \alpha_{1,\text{loc})}} \\
(F_a - F_b) R_1 = C_1
\]

- Case $\tau > 1$

\[
F_a = q\omega_1^2 R_1 + (F_b - q\omega_1^2 R_1) \ e^{(\omega_1 - \alpha_{1,\text{loc})}} \\
F_a = q\omega_2^2 R_2 + (F_b - q\omega_2^2 R_2) \ e^{\omega_2 t} \\
(F_a - F_b) R_1 = C_1
\]

where

\[
V_i = \tan \gamma_0 \left. \frac{dr}{dt} \right|_{R_b} + r_0 \omega_i \\
\left(1 + \frac{F_b}{EA} \right) \left(\tan \gamma_0 \sin \theta_0 + \cos \theta_0 \right),
\]

for $i = 1, 2$.

If the values of $V_1$ and $V_2$ are too much different, return to step 4 maintaining the updated value of $V$. The new value of $v$ is computed again by means of (9), but with the updated values of $V$ and $F = F_b$.

10. By varying the value of $\alpha_{i,\text{loc}}$ (the subscript $i = 2$ when $\tau < 1$ and vice versa), the quantity is minimized

\[
g(\alpha_{i,\text{loc}}) = \int_{\alpha_{1,\text{loc}}}^{\alpha_{2,\text{loc}}} \frac{dF}{d\phi} d\phi - \int_{\alpha_{2,\text{loc}}}^{\alpha_{1,\text{loc}}} \frac{dF}{d\phi} d\phi
\]

and all subsequent quantities updated starting from step 3.

The procedure is stopped when the difference (27) between the computed forces $F_b$ for both pulleys are small.

Under transient conditions the computed value of $\alpha_{i,\text{loc}}$ can be negative. If this is the case, then we let $\alpha_{i,\text{loc}} = 0$ and the iteration is repeated, after changing the value of the subscript $i$.

11. At this step, $F_a$ and $F_b$ and $\alpha_{1,\text{loc}}$ are known.

However, the value of $F_a$ and subsequently all the others, need to be refined through a new iteration.

A new iteration from step 3 is started until the equation

\[
f(F_a) \equiv \left( \frac{1}{\omega_i} \int_{\alpha_{1,\text{loc}}}^{\alpha_{2,\text{loc}}} 2\mu \frac{dN}{d\phi} \frac{v_g}{\cos \theta_i} \ d\phi \right) - \left( C_1 - \int_{\alpha_{1,\text{loc}}}^{\alpha_{2,\text{loc}}} \frac{dF}{d\phi} r d\phi \right)
\]

is satisfied.

At the end of this step the values of $F_a$, $F_b$ and $\alpha_{i,\text{loc}}$ are accepted.

12. Compute output torque

\[
C_2 = \int_{\alpha_{2,\text{loc}}}^{\alpha_{1,\text{loc}}} \frac{dF}{d\phi} r d\phi
\]

and mechanical efficiency

\[
\eta = \left| \frac{C_2 \omega_2}{C_1 \omega_1} \right|
\]
5 A brief description of the experimental setup

The experimental part has been carried out Centro Ricerche Fiat of Orbassano (TO). Figure 11 gives an overview of the scheme of the experimental setup. Both the CVT and the control unit tested are commercially available.

6 Results

In this section the simulation results are discussed. For the simulation the following numerical data have been used (SI units): \( f=0.05, \quad q=1.2, \quad h=0.146, \quad I_b=0.628, \quad \beta_0=10.8 \text{ deg}, \quad E A=8.24 \times 10^6, \quad I_2=10. \) The CVT is shifting in a traditional way (discrete speed ratios). In particular we started from the sixth gear and shifted down till first gear and then shifted up to sixth gear again. (See figure 12) The figures 13 and 14 show the differences between the computed and measured output torques. All the values have been normalized. The differences are likely due to the strategy adopted by the control unit and not included in our model. In fact, to avoid sliding and allow a full torque transmission at the different working conditions, the control may increase the pressure on the pulley. Consequently the measured transmitted torque is higher than the one theoretically predicted. The difference between computed and experimental mechanical efficiency is shown in figure 15. These differences may be explained by considering that losses due to ventilation, friction on bearings are not included in this model. In fact, considering also such losses consistently with the estimate of (Sattler, 1999a), the differences are shrunk.

7 Conclusions

A model for the simulation of CVT under transient behaviour has been presented.

The method used is an extension of the work described in (Sattler, 1999a). The basic model has been improved introducing inertia and other transient effects. A procedure to take in account pulley deformation has also been embodied in the model. Discrete and continuous shifting have been simulated in order to
compute efficiency and power losses due to friction between belt and pulley halves. The results have shown high loss of efficiency during transient of shifting. Power losses due to frictional effects not included (e.g. bearing, churning losses, etc) could be added to the model proposed after experimental tests. The proposed model has been experimentally validated on a commercial type CVT.

![Graph 13](image13.png)
**Figure 13.** Input torque prescribed and output torque measured

![Graph 14](image14.png)
**Figure 14.** Input torque prescribed and output torque computed

**REFERENCES**


Sferra, D., *Analisi in Condizioni Stazionarie e Transitorie del Meccanismo di Trasmissione della Coppia in un Cambio Continuo con Cinghia Van Doorne per uso Automobilistico*, Thesis in Mechanical Engineering, University of Tor Vergata, December 2001

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