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Theoretical and Experimental Efficiency Analysis of Multi-Degrees-of-Freedom Epicyclic Gear Trains

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Abstract. Mechanical efficiency analysis is a fundamental phase in the design of Power Gear Trains (PGTs) transmissions. This paper presents a methodology for computing the mechanical efficiency of Epicyclic Gear Trains (EGTs). A novel feature of this methodology is the capability to take into account load dependent power losses and inertia effects. Finally, the paper describes an apparatus used for the experimental validation of the proposed methodology.

Key words: gear trains, mechanical efficiency.

Nomenclature

$\mathrm{d}s$	=	infinitesimal arc length of gear centrodes
$\{F^a\}$	=	vector of generalized forces due to friction
$\{F_e\}$	=	vector of generalized external forces
F_{ij}	=	tangent component of meshing force exherted by wheel i on wheel j
M_m, M_u	=	driving and resisting torques acting, respectively, on the wheels of an elementary gear train unit
$[M]$	=	mass matrix
N_d	=	number of driving constraints
N_o	=	number of output shafts
$\{\lambda\}$	=	vector of Lagrange's multipliers
p	=	gears circular pitch
$\{q\}$	=	vector of generalized coordinates
r_i	=	radius of the pitch circle of wheel i
R_{ij}	=	reaction force exerted by body i on body j
T_d, T_o	=	driving and load torques, respectively
z_i	=	number of gear teeth of wheel i
δ	=	gear reaction force offset due to friction
ϕ	=	friction angle
$\{\Psi\}$	=	vector kinematic constraints

η = average mechanical efficiency
ϑ = pressure angle
ρ_i = friction radius of gear wheel i
ω_d, ω_o = absolute angular velocities of driving and driven links, respectively
ω_{ik} = relative angular speed of link i w.r.t. link k

Sub-, superscripts
$_m$ = the average value
$\hat{}$ = the algebraic sign of the versor
$\dot{}$ = differentiation w.r.t. time
* = computed values

1. Introduction

Experimental evidence shows that the mechanical efficiency of gear trains is sensitive to load. Within a limited range, this efficiency increases with load. Moreover, examination of transients of gear trains is significant for the overall efficiency analysis of city vehicles. Such analysis requires also the consideration of lubrication conditions and inertia effects.

With reference to the computing of mechanical efficiency in Epicyclic Gear Trains (EGT), a review of the published literature reveals that most of the available methods (e.g. [1–4]):

- consider only the kinematics of the gear train;
- for a fixed value of the friction coefficient, the computed value of the mechanical efficiency is not load dependent;
- angular velocities are constant;
- bearing losses are not included.

The described situation motivated the authors to propose a new method for mechanical efficiency analysis of spur-gears EGTs.

It is well known that an EGT can be assimilated to an assembly of elementary gear trains units.

The units are composed of two gears and one gear carrier connecting them (Figure 1). Their arrangement is said ordinary (epicyclic) when the gear carrier is fixed (rotating). Many reliable methods are available for computing the mechanical efficiency of an ordinary gear train unit.

For an elementary epicyclic gear train unit, by means of a *kinematic inversion*,⋆ it is herein suggested a way to extend these methods for computing the torques introduced by the dissipative effects.

⋆ The procedure of changing the choice of link to be fixed while leaving the kinematic chain unchanged is called *kinematic inversion*. Analytically this is obtained subtracting, from the absolute angular velocities of the bodies, the absolute angular velocity of the link to be fixed. Inversion will not result in any alteration of relative motions of the bodies in a mechanism.

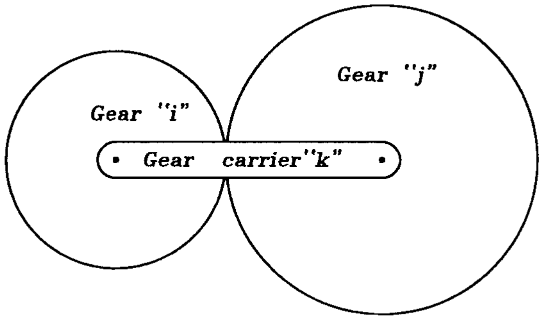

Figure 1. Elementary gear train unit.

This approach, combined with a multibody technique, allows a systematic mechanical efficiency analysis of EGTs with complex topologies.

In the following sections the following will be discussed:

- the multibody formalism used for solving the inverse dynamic problem in gear trains;
- the model for computing the average efficiency of elementary ordinary gear trains units;
- the proposed method for computing the torques due to friction in any elementary gear train unit;
- the experimental setup.

Finally, a comparison between theoretical and experimental validated the method of analysis proposed.

2. Inverse Dynamic Analysis by Means of a Multibody Formalism

When solving the inverse dynamic problem, the values of accelerations, velocities and positions are imposed and the magnitude and directions of the generalized forces, required to provide the desired motion, are unknowns.

In our approach, the kinematic analysis is carried out by means of the constraint method. Since a detailed account of the method is available in several textbooks (e.g. [7–10]), will not be repeated here.

According to this method, position, velocity and acceleration analysis require, respectively, the solution of the following systems of equations

$$\{\Psi\} = \{0\}, \tag{1}$$

$$[\Psi_q]\{\dot{q}\} = \{-\Psi_t\}, \tag{2}$$

$$[\Psi_q]\{\ddot{q}\} = -([\Psi_q]\{\dot{q}\})_q\{\dot{q}\} - 2[\Psi_{qt}]\{\dot{q}\} - \{\Psi_{tt}\}, \tag{3}$$

where q is the vector of generalized coordinates.

For the planar motion, the position of the ith body is specified by means of three coordinates:

- q_{3i-2}, q_{3i-1}: absolute Cartesian coordinates of the center of mass;
- q_{3i}: absolute angular position of the body.

The constraint equations can be split into two classes: scleronomic and rheonomic (driving) constraints. The number of rheonomic constraints is equal to the degrees-of-freedom of the gear train. Moreover, we assume that there is no kinematically redundant link, thus the Jacobian matrix $[\Psi_q]$ has always full rank.

The equilibrium equations for all bodies of a mechanical system can be written in the following form [11]

$$[M]\{\ddot{q}\} + [\Psi_q]^T\{\lambda\} = \{F_e\} + \{F^a(q, \dot{q}, \lambda)\}. \tag{4}$$

For the solution of the inverse dynamics problem, is useful to rearrange the previous system as follows

$$[\Psi_q]^T\{\lambda\} = -[M]\{\ddot{q}\} + \{F_e\} + \{F^a(q, \dot{q}, \lambda)\}, \tag{5}$$

where the unknown torques T_{di} $(i = 1, \ldots, N_d)$ follow from the components of vector λ associated with the rheonomic constraints. (An example which clarifies this statement is discussed in the Appendix.)

At constant angular speed, under the hypothesis of balanced gears and gear carriers, the term $-[M]\{\ddot{q}\}$ representing generalized inertia forces can be omitted.

When friction effects are included, the previous system is iteratively solved by starting with the value of λ obtained for $F^a = 0$.

Once the output torques T_{oj} $(j = 1, \ldots, N_o)$ are prescribed and the input torques T_{di} $(i = 1, \ldots, N_d)$ computed, the average mechanical efficiency $\eta^\star$ follows from

$$\eta^\star = \frac{\sum_{j=1}^{N_o} T_{oj}\omega_{oj}}{\sum_{i=1}^{N_d} T_{di}\omega_{di}}. \tag{6}$$

3. The Average Mechanical Efficiency of an Ordinary Gear Train

With reference to Figures 2 and 3, because of the presence of friction, according to the well known Buckingham's model [22], the direction of the meshing force intersects the center distance in a point P, which is not the center of instantaneous rotation P_0. Let $\delta = P_0P$ be the distance between such points. During teeth meshing this distance is not constant.

Under the hypotheses of only one tooth in contact and considering meshing losses only, the instantaneous mechanical efficiency η of an ordinary gear train can be computed by means of the following formulas:

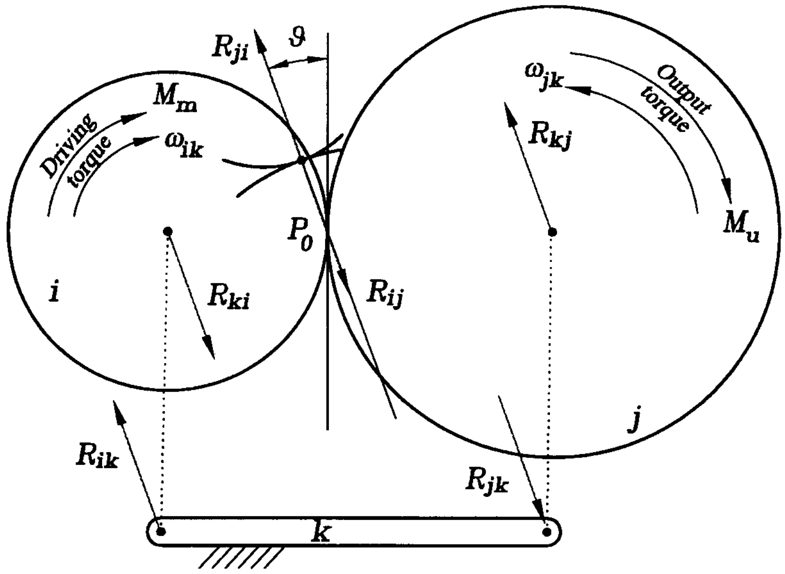

Figure 2. Forces in an ordinary gear train unit without friction.

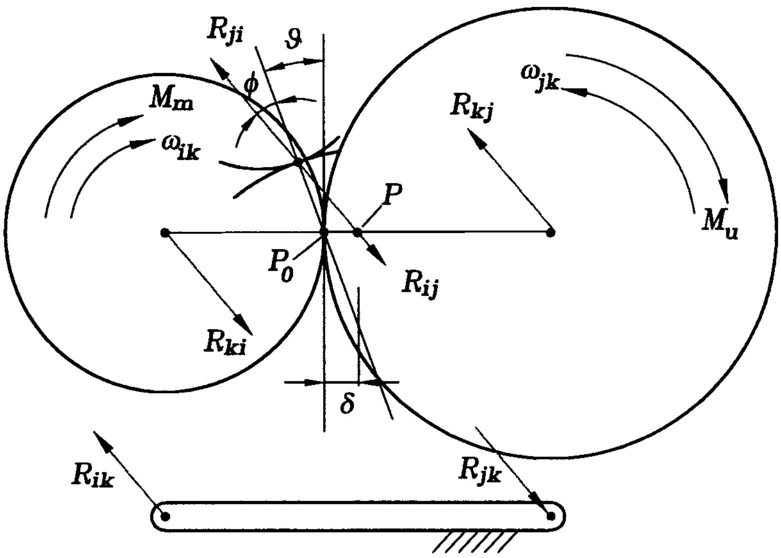

Figure 3. Forces in an ordinary gear train unit with friction at contact of teeth only.

- External gears
  - Driving wheel: i; driven wheel: j

$$\eta = \frac{1 - \frac{\delta}{r_j}}{1 + \frac{\delta}{r_i}}. \tag{7a}$$

  - Driving wheel: j; driven wheel: i

$$\eta = \frac{1 - \frac{\delta}{r_i}}{1 + \frac{\delta}{r_j}}. \tag{7b}$$

- Internal gears
  - Driving wheel: i; driven wheel: j

$$\eta = \frac{1 - \frac{\delta}{r_j}}{1 - \frac{\delta}{r_i}}. \tag{7c}$$

  - Driving wheel: j; driven wheel: i

$$\eta = \frac{1 - \frac{\delta}{r_i}}{1 - \frac{\delta}{r_j}}. \tag{7d}$$

Let us denote with

$$\eta_m = \frac{\int_p \eta \, \mathrm{d}s}{p} \tag{8}$$

the average mechanical efficiency of an ordinary two gear train and with δ_m the average value of δ.

Since the value of η_m can be computed by means of one of the methods reported in literature (e.g. [12–14]) or, alternatively, estimated from experimental data, an approximation of δ_m is deduced from the previous expressions by setting in (7) $\eta = \eta_m$ and solving w.r.t. δ_m.

Thus, the following formulas are, respectively, obtained:

- External gears

  – Driving wheel: i; driven wheel: j

$$\delta_m = \frac{(1-\eta_m) r_i r_j}{\eta_m r_j + r_i}. \tag{9a}$$

  – Driving wheel: j, driving wheel: i

$$\delta_m = \frac{(1-\eta_m) r_i r_j}{\eta_m r_i + r_j}. \tag{9b}$$

- Internal gears

  – Driving wheel: i, driven wheel: j

$$\delta_m = \frac{(\eta_m - 1) r_i r_j}{\eta_m r_j - r_i}. \tag{9c}$$

  – Driving wheel: j, driving wheel: i

$$\delta_m = \frac{(\eta_m - 1) r_i r_j}{\eta_m r_i - r_j}. \tag{9d}$$

3.1. THE MODEL OF ANDERSON AND LOWENTHAL

Expressions (9) show that an estimate of the average efficiency of an ordinary gear train η_m is required for the computation of δ_m.

For this task, the authors adopted the approximate method of Anderson and Lowenthal [12]. The method, based on power loss expressions evaluated at only one point along the path of contact, account for sliding, rolling, bearing and windage losses.

The gear data required by the model are the following:

– number of gear teeth;
– diametral pitch;
– gear ratio;
– gear width;
– pressure angle ϑ;
– bearing pitch diameter;
– pinion angular speed;
– pinion torque;

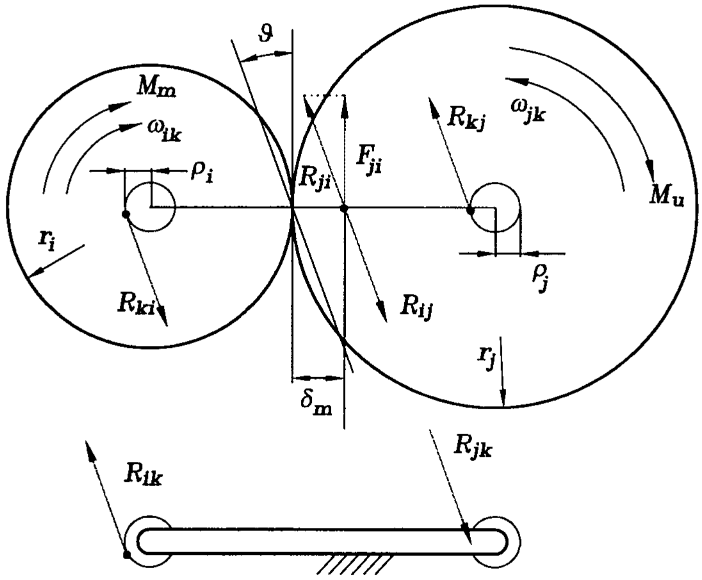

Figure 4. Refined model for computing the effects of friction in an ordinary gear train unit (friction in revolute pairs has been added).

- lubricant kinematic and absolute viscosities;
- ball-bearing lubrication factor;
- support-bearing basic static capacity;
- static equivalent bearing load.

A more sophisticated model, due to Anderson and Lowenthal [14], for the computation of the equivalent ordinary gear drive is available. However, this requires a numerical integration along all the path of teeth contact.

The epicyclic gear arrangement is reduced to an ordinary gear train by means of a kinematic inversion which makes fixed the gear carrier [1–3]. Thus, formulae (7) and (9), valid for the ordinary, can be extended to the elementary gear trains with epicyclic arrangement.

4. Computation of Torques due to Friction

For the generic nth gear train unit, the dynamic effects of friction are reduced to three torque components $F_{3i}^{a_n}$, $F_{3j}^{a_n}$, $F_{3k}^{a_n}$, acting on the gear wheels i and j and gear carrier k, respectively.

With reference to the nomenclature of Figure 4, the proposed computational scheme of such torques is given below:

1. The kinematic constraint for the nth gear pair is the Willis' equation

$$\Psi_n \equiv r_i(q_{3i} - q_{3i}^0) - r_j(q_{3j} - q_{3j}^0) + (r_j - r_i)(q_{3k} - q_{3k}^0) = 0, \tag{10}$$

   where ^{0} denotes the initial values of coordinates.
2. Let $|\lambda_n|$ the Lagrange's multiplier associated with the Willis' constraint equation. This represents the tangent component F_{ij} of the meshing force R_{ij}. When there is no friction, we have

$$F_{ij} = R_{ij} \cos \vartheta. \tag{11}$$

   For small values of the friction angle ϕ, we can assume

$$F_{ij} \approx R_{ij} \cos \vartheta. \tag{12}$$

3. Compute the value of η_m by means of the model of Lowenthal and Anderson. For this purpose, a kinematic inversion on the gear train unit is applied. Under this inversion the gear carrier k become fixed and the elementary gear train unit is transformed from epicyclic into ordinary.
   Once the value of η_m is obtained, among the expressions (9), the appropriate one is chosen for computing δ_m.
4. Denote with $\hat{\omega}_{ik}$, $\hat{\omega}_{jk}$ the algebraic sign of the relative angular velocities of gear wheels i and j w.r.t. the gear carrier k.
5. According to our model, the friction introduces the following resisting torque components on the links of the elementary gear train unit considered:

   – Friction torque on wheel i

$$F_{3i}^{a_n} = -F_{ij}\delta_m\hat{\omega}_{ik} - \frac{F_{ij}}{\cos\vartheta}\rho_i\hat{\omega}_{ik}. \tag{13a}$$

   – Friction torque on wheel j

$$F_{3j}^{a_n} = -F_{ij}\delta_m\hat{\omega}_{jk} - \frac{F_{ij}}{\cos\vartheta}\rho_j\hat{\omega}_{jk}. \tag{13b}$$

   – Friction torque on the gear carrier k

$$F_{3k}^{a_n} = \frac{F_{ij}}{\cos\vartheta}\rho_i\hat{\omega}_{ik} + \frac{F_{ij}}{\cos\vartheta}\rho_j\hat{\omega}_{jk}. \tag{13c}$$

6. Compute (13) for all the elementary gear units of the EGT.

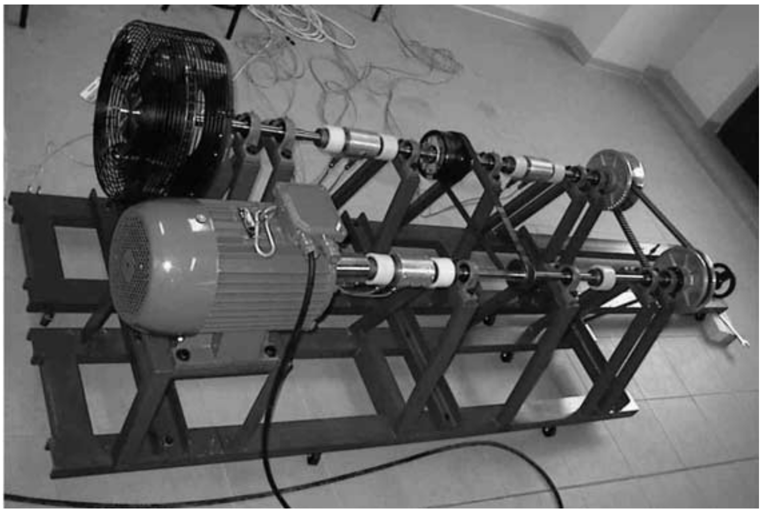

Figure 5. View of the experimental setup.

The generalized force vector introduced by friction is computed through the sum

$$\{F^{a}\} = \sum_{n} F^{a_n}, \tag{14}$$

where

$$\{F^{a_n}\} = \begin{Bmatrix} \vdots \\ F_{3i}^{a_n} \\ \vdots \\ F_{3j}^{a_n} \\ \vdots \\ F_{3k}^{a_n} \\ \vdots \end{Bmatrix} \tag{15}$$

is the friction torque vector due to the nth gear pair.

This vector is substituted into (5) and the system solved with respect to $\{\lambda\}$. The entire procedure is iterated until convergence is achieved. Usually 2–3 iterations only are required.

5. The Experimental Setup

In previous papers [18, 19] an original Power Split CVT system was proposed with two separate phases of operation able to guarantee a power flow without circulation.

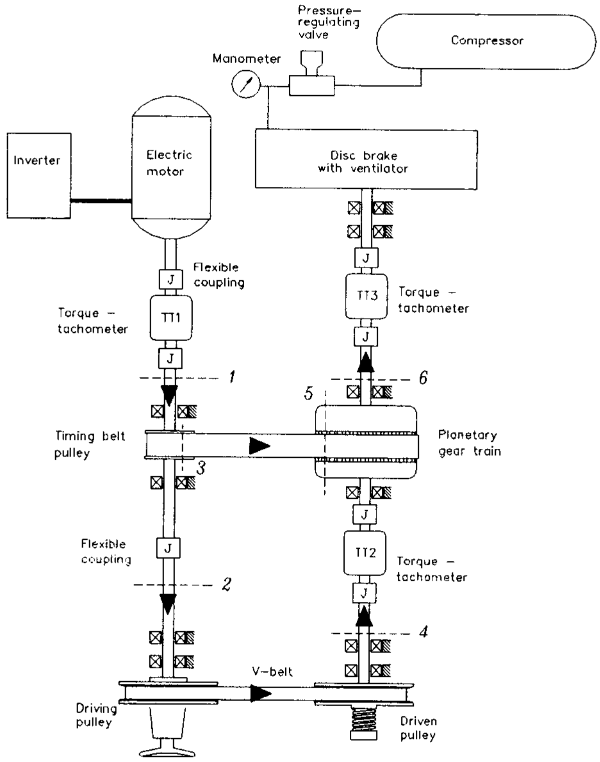

Figure 6. Scheme of the test rig used.

A special test rig (Figure 5) was built especially for measurement of power flows and efficiency in Power Split CVT planetary gear train. Within the same speed ratio range, this arrangement has a better efficiency when compared to a traditional CVT.

In other papers [20, 21] a theoretical and experimental study regarding the performances of Infinitely Variable Transmissions was performed. The test rig was used to measure the efficiency of the planetary gear train under analysis. The Power Split CVT system allowed the regulation of the relative angular speeds among links.

In Figure 6 the scheme of the test rig used in this investigation is shown.

The main components of the Power Split CVT system (PS-CVT) are the following:

- a Planetary Gear Train (PG);
- a Continuously Variable Transmission (CVT);
- a Fixed Speed Ratio mechanism (FR).

The PS-CVT was built using a CVT (Gerbes RF 210b) with a rubber V-belt. Since steady state operating conditions are monitored, the variation of transmission ratio was carried out by using a knob that regulates the position of the mobile driving half-pulley.

For the test rig, a commercial planetary gear train (Andantex SR20) was used. The motion of the gear carrier was regulated by a timing belt (type L) for which the constant speed ratio transmission (FR) was obtained by means of a timing belt pulley.

The torque and the angular speed of the branches of the PS-CVT were measured by a torque-tachometer (model TT/9000 – 50 Nm – Tekkal). Two torque-tachometers were used, one to measure the output (TT3) power of the planetary gear and one placed between the planetary gear and the CVT (TT2). Each torque-tachometer was connected to the shafts through flexible coupling (J) in order to avoid any bending moments that would invalidate the measurement. The test rig was powered by an AC motor with three magnetic poles driven through an Inverter (Gerbes ACM Compact 5.5 kW).

This allows tests at different input speeds. The brake system was obtained by means of a pneumatic disk brake with forced ventilation. A pressure regulating valve, by altering the input pressure of the brake, allowed to change the output torque of the PS-CVT system. The signals from the torque-tachometers were treated by an amplifier and feeded into a PC for acquisition and storage. The experimental results of the efficiency of the planetary gear train are found for an input frequency of the AC motor of 25 Hz (500 rpm) in order to vary the output torque. The tests were performed for different speed ratios (0.4, 0.8, 1.2, 1.6, 2, and 2.3) of the CVT. The torque T_{d_2} acting on the gear carrier for the PGT was calculated using the equation of the rotation equilibrium

$$T_o + T_{d_1} + T_{d_2} = 0, \tag{16}$$

in which T_{d_1} and T_o are the torque measured, respectively, by the torque-tachometers TT2 and TT3.

The epicyclic gear train under test is the model SR 20 manufactured by Andantex. The kinematic structure of this two-degrees-of-freedom gearbox is shown in Figure 7. The number of teeth of the gear wheels are as follows: $z_1 = 36$, $z_2 = 32$, $z_3 = 20$, $z_4 = 45$.

6. Validation of the Theoretical Model

In the numerical application the following data are assumed: The numerical data specifying the characteristics of the gear pairs in the EGT analyzed are as follows:

1. First gear pair: number of teeth: 36 and 32, diametral pitch = 425 m^{-1}.
2. Second gear pair: number of teeth: 20 and 45, diametral pitch = 406.25 m^{-1}.

- bearings bore diameter: $D_m = 0.03$ m;

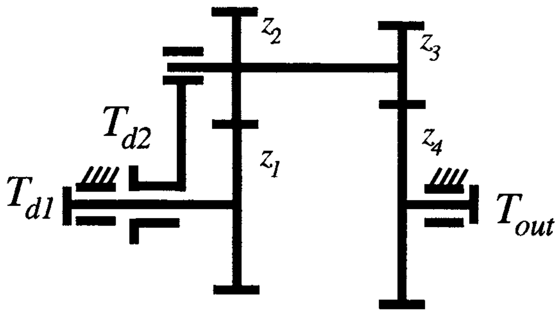

Figure 7. Kinematic structure of the EGT analyzed.

- pressure angle: $\theta = 20$ deg;
- teeth width: $\mathcal{F} = 0.017$ m;
- lubricant kinematic viscosity: $v_B = 1$ cm/s^2;
- lubricant absolute viscosity: $\mu_0 = 0.09$ Ns/m^2;
- support-bearing basic static capacity: $C_s = 20000$ N;
- ball-bearing lubrication factor: $f_0 = 2$.

Since the method of Anderson and Lowenthal already includes ball bearing losses, in Equations (13) the radius ρ of friction circle is set equal to zero.

6.1. COMPUTATION OF THE MECHANICAL EFFICIENCY BY MEANS OF EXPERIMENTAL VALUES

- Set of variables measured: T_o, T_{d_1}, ω_o, ω_{d_1}, ω_{d_2}.
- Set of variables obtained from measured data:

$$T_{d_2} = -(T_{d_1} + T_o), \tag{17}$$

$$\eta_{PGT} = \left| \frac{T_o \omega_o}{T_{d_1}\omega_{d_1} + T_{d_2}\omega_{d_2}} \right|. \tag{18}$$

6.2. COMPUTATION OF THE MECHANICAL EFFICIENCY BY MEANS OF THE PROPOSED MODEL

- Set of variables prescribed (same as measured data): T_o, T_{d_1}, ω_{d_1}, ω_{d_2}.
- Angular velocity computed: ω_o;
- Torques computed ($N_d = 2$): $\Delta T^\star_{d_1}$, $T^\star_{d_2}$. In particular, these torques follow from the values of Lagrage's multipliers associated with the driving constraints imposed at the driving shafts d_1 and d_2, respectively.

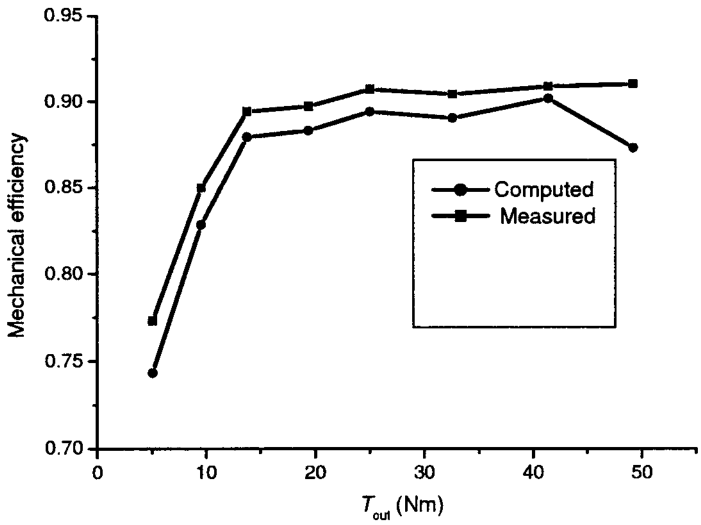

Figure 8. Comparison of mechanical efficiencies (see Table I for the other parameters).

Table I. First set of working conditions.

T_o (Nm)	$T^{\star}_{d_2}$ (Nm)	T_{d_1} (Nm)	$\Delta T^{\star}_{d_1}$ (Nm)	ω_o (r.p.m.)	ω_{d_1} (r.p.m.)	ω_{d_2} (r.p.m.)	$\eta^{\star}_{\text{PGT}}$	η_{PGT}
5.08	–2.43	–3.58	0.926	180	309	51	0.7433	0.7731
9.56	–4.62	–5.96	1.01	180	309	51	0.8284	0.8497
13.8	–6.69	–8.04	0.926	180	309	51	0.8791	0.8941
19.43	–9.44	–11.25	1.25	179	309	49	0.8830	0.8970
25.05	–12.182	–14.31	1.43	178	306	50	0.8939	0.9070
32.6	–15.86	–18.72	1.978	178	305	51	0.8902	0.9042
41.42	–20.157	–23.45	2.18	177	302	52	0.9017	0.9086
49.2	–23.944	–28.175	2.91	174	299	49	0.8731	0.9102

The computed torques at the driving shafts d_1 and d_2 are, respectively, $T^{\star}_{d_1} = T_{d_1} + \Delta T^{\star}_{d_1}$ and $T^{\star}_{d_2}$. In the evaluation of the mechanical efficiency it was assumed that $T^{\star}_{d_1} \approx T_{d_1}$. This approximation is supported by the following considerations:

– T_{d_1} and T_o are directly measured with torquetachometers TT2 and TT3 and can be considered as data of our problem;

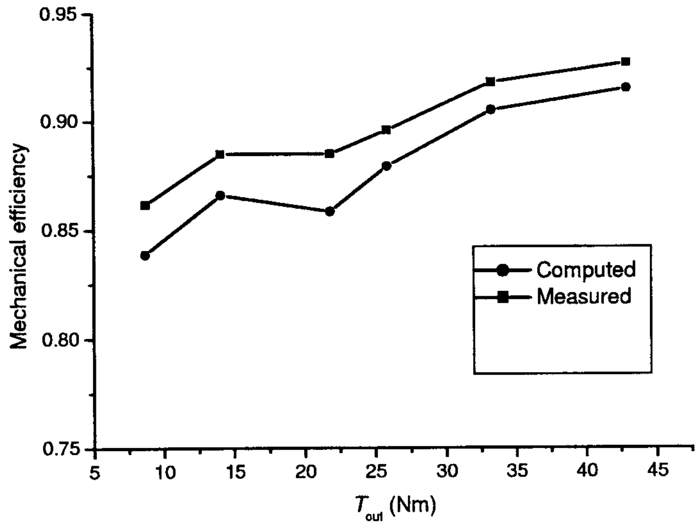

Figure 9. Comparison of mechanical efficiencies (see Table II for the other parameters).

Table II. Second set of working conditions.

T_o (Nm)	$T^{\star}_{d_2}$ (Nm)	T_{d_1} (Nm)	$\Delta T^{\star}_{d_1}$ (Nm)	ω_o (r.p.m.)	ω_{d_1} (r.p.m.)	ω_{d_2} (r.p.m.)	$\eta^{\star}_{\mathrm{PGT}}$	η_{PGT}
8.68	–4.19	–5.36	0.868	184	310	58	0.8385	0.8613
14.06	–6.82	–8.36	1.117	183	309	57	0.8657	0.8847
21.83	–10.61	–13.28	2.057	182	308	56	0.8581	0.8956
25.87	–12.581	–15.1	1.807	181	307	55	0.8789	0.8956
33.30	–16.202	–18.8	1.698	180	305	55	0.9047	0.9177
42.92	–20.885	–23.95	1.91	179	301	57	0.9147	0.9267

– A close matching between $\eta^{\star}_{PGT}$ and η_{PGT} was obtained when $\Delta T^{\star}_{d_1}$ was neglected.

Therefore, the equation used for computing the mechanical efficiency was the following:

$$\eta^{\star}_{PGT} = \left| \frac{T_o \omega_o}{T_{d_1}\omega_{d_1} + T^{\star}_{d_2}\omega_{d_2}} \right|. \tag{19}$$

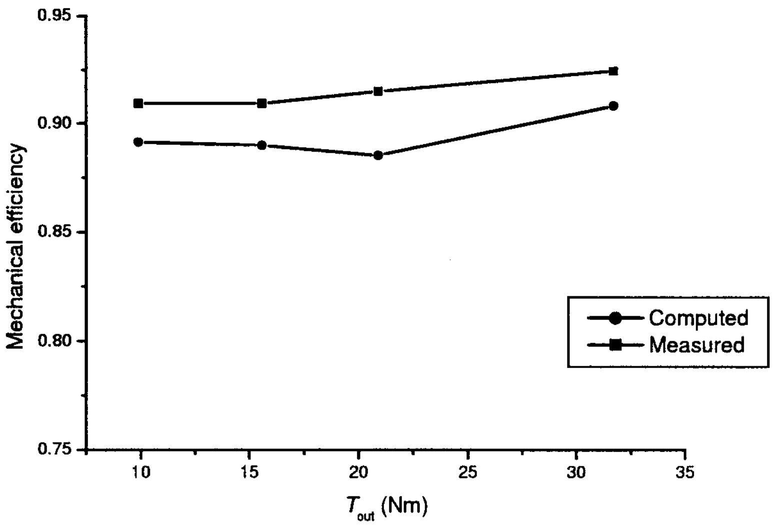

Figure 10. Comparison of mechanical efficiencies (see Table III for the other parameters).

Table III. Third set of working conditions.

T_o (Nm)	$T^{\star}_{d_2}$ (Nm)	T_{d_1} (Nm)	$\Delta T^{\star}_{d_1}$ (Nm)	ω_o (r.p.m.)	ω_{d_1} (r.p.m.)	ω_{d_2} (r.p.m.)	$\eta^{\star}_{\text{PGT}}$	η_{PGT}
9.89	–4.77	–5.72	0.60	402	658	146	0.8914	0.9092
15.57	–7.55	–9.02	1.00	400	652	148	0.8898	0.9091
20.9	–10.17	–12.18	1.45	392	638	146	0.8852	0.9147
31.69	–15.45	–17.9	1.66	374	610	138	0.9081	0.9243

6.3. NUMERICAL RESULTS

The mechanical efficiency of the epicyclic gear train has been measured and theoretically estimated with two different set of working parameters.

With reference to the working conditions reported in Tables I–V, Figures 8–12 show a comparison between the computed and measured mechanical efficiencies of the epicyclic gear train under investigation.

7. Conclusions

A new method for predicting the efficiency of spur gears epicyclic drive has been proposed. The method accounts for losses due to sliding, rolling, windage and ball bearings. The method was validated by comparing the theoretical results with test data under different load and kinematic conditions.

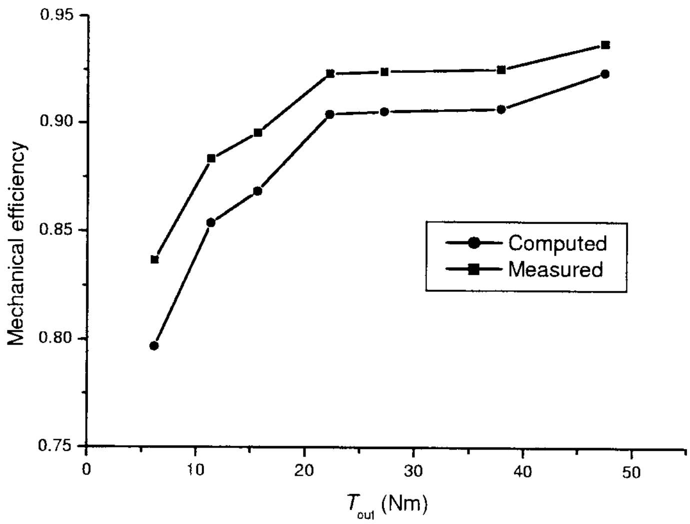

Figure 11. Comparison of mechanical efficiencies (see Table IV for the other parameters).

Table IV. Fourth set of working conditions.

T_O (Nm)	$T^{\star}_{d_2}$ (Nm)	T_{d_1} (Nm)	$\Delta T^{\star}_{d_1}$ (Nm)	ω_o (r.p.m.)	ω_{d_1} (r.p.m.)	ω_{d_2} (r.p.m.)	$\eta^{\star}_{PGT}$	η_{PGT}
6.16	–2.96	–4.10	0.90	195	310	80	0.7966	0.8366
11.33	–5.49	–6.93	1.08	194	309	79	0.8536	0.8831
15.58	–7.56	–9.34	1.32	194	308	80	0.8681	0.8953
22.16	–10.77	–12.64	1.25	193	307	79	0.9039	0.9232
27.15	–13.20	–15.45	1.50	192	306	78	0.9054	0.9242
37.84	–18.40	–21.50	2.06	190	302	78	0.9068	0.9256
47.38	–23.04	–26.32	1.98	188	298	78	0.9240	0.9374

Noteworthy features of the proposed method are:

- Systematicity of the approach.

  The method described can be applied to multi-degrees-of-freedom spur gear epicyclic gear trains with complex kinematic structures.
- A correct prediction of mechanical efficiency variation due to changes of load and speed.

  Although a close correspondence between the measured and predicted values of efficiency is not always achieved, the differences between computed and measured values are in most of the cases acceptable for engineering purposes.

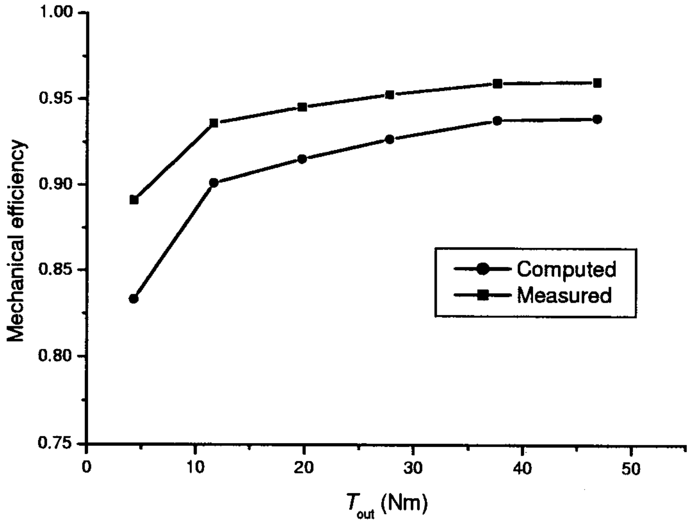

Figure 12. Comparison of mechanical efficiencies (see Table V for the other parameters).

Table V. Fifth set of working conditions.

T_O (Nm)	$T^{\star}_{d_2}$ (Nm)	T_{d_1} (Nm)	$\Delta T^{\star}_{d_1}$ (Nm)	ω_o (r.p.m.)	ω_{d_1} (r.p.m.)	ω_{d_2} (r.p.m.)	$\eta^{\star}_{PGT}$	η_{PGT}
4.31	–2.06	–2.81	0.56	251	352	150	0.8334	0.8910
11.64	–5.64	–6.81	0.81	249	349	149	0.9009	0.9360
19.74	–9.58	–11.30	1.14	246	344	148	0.9152	0.9454
27.73	–13.47	–15.60	1.34	243	339	147	0.9270	0.9529
37.60	–18.27	–20.81	1.48	239	333	145	0.9381	0.9596
46.76	–22.72	–25.84	1.80	235	327	143	0.9392	0.9604

The authors believe that such differences could be lowered through the adoption of more accurate models for computing:

– bearing losses;
– meshing losses;
– churning and windage losses.

The oil temperature is also a factor that influenced the reported discrepancies between measured and computed quantities. In fact, during this investigation, the oil temperature and change in viscosity were not monitored during the experiments.

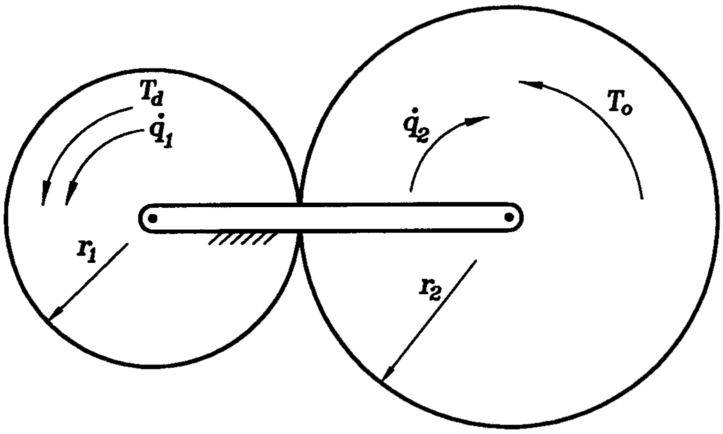

Figure 13. Ordinary gear train.

Appendix

In the ordinary gear train, shown in Figure 13, wheel 1 is required to rotate at constant angular speed ω_1. Thus, the following constraints can be imposed:

$$\Psi_1 \equiv q_1 - \omega_1 t = 0, \tag{20}$$

$$\Psi_2 \equiv r_1(q_1 - q_1^0) - r_2(q_2 - q_2^0) = 0. \tag{21}$$

For simplicity the constraints due to the revolute pairs are omitted and angles are the only generalized coordinates.

Under the hypothesis of absence of friction, from Equation (5) one readily obtains

$$\begin{bmatrix} 1 & r_1 \\ 0 & r_2 \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix} = - \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ T_o \end{Bmatrix}, \tag{22}$$

where I_i ($i = 1, 2$) denote the moment of inertia of the gears.

This elementary example, adapted from [10], show that the Lagrange's multiplier:

- λ_1, associated with the rheonomic constraint, corresponds to the driving torque T_d;
- λ_2, associated with the Willis' equation, corresponds to the tangent component of the meshing force.

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