# Strategies for the Numerical Integration of DAE Systems in Multibody Dynamics

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**ABSTRACT:** The number of multibody dynamics courses offered in the university is increasing. Often the instructor has the necessity to go through the steps of an algorithm by working out a simple example. This gives the student a better understand of the basic theory. This paper provides a tutorial on the numerical integration of differential-algebraic equations (DAE) arising from the dynamic modeling of multibody mechanical systems. In particular, some algorithms based on the orthogonalization of the Jacobian matrix are herein discussed. All the computational steps involved are explained in detail and by working out a simple example. It is also reported a brief description and an application of the multibody code NumDyn3D which uses the Singular Value Decomposition (SVD) approach.
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## INTRODUCTION

The multibody formulation discussed in many multibody dynamics textbooks (e.g., [1–3]) requires the position of each rigid body to be described by means of seven generalized coordinates: three for the position ($x$, $y$, $z$) and the four Euler's parameters to define spatial orientation ($e_0$, $e_1$, $e_2$, $e_3$) of a body. As a consequence the set of coordinates is redundant, i.e., the number of coordinates is higher than the degrees-of-freedom (d.o.f.) of the mechanical system under simulation. This leads to a DAE system which can be solved only numerically. In order to reduce the DAE system to an ordinary differential equations (ODE) the orthogonalization of the Jacobian matrix of the constraint equations is necessary. This approach offers the following advantages:

1. The elimination of Lagrange's multipliers when solving equations;
2. The possibility to partition the entire set of generalized coordinates into independent variables and dependent ones;
3. The transform of the DAE system into a ODE gives the opportunity of a wider choice of numerical integration subroutines;
4. Mechanical systems with a redundant number of constraints or with changing d.o.f. can be analysed.

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## NOMENCLATURE

- $F$ degrees-of-freedom of the mechanical system under analysis;
- $\{F_e\}$ is the vector of generalized applied forces;
- $\{Fe^p\}$ is the vector of generalized applied forces related to independent coordinates;
- $Fe^u$ is the vector of generalized applied forces related to dependent coordinates;
- $\{\hat{F}(t,p,\dot{p})\}$ is the vector of generalized applied forces of the SSODE system;
- $I$ is the identity matrix;
- $[M]$ is the symmetric $n \times n$ system mass matrix;
- $[M^{pp}]$ is the $F \times F$ mass sub-matrix related to independent coordinates;
- $[M^{uu}]$ is the $(n-F) \times (n-F)$ mass sub-matrix related to dependent coordinates;
- $[M^{pu}]$ is the $F \times (n-F)$ mass sub-matrix;
- $[M^{up}]$ is the $(n-F) \times F$ mass sub-matrix;
- $[\hat{M}(p)]$ is the mass matrix of the SSODE system;
- $m$ number of constraint equations;
- $n$ number of generalized coordinates;
- $\{p\}$ a vector of $F$ independent coordinates;
- $q = \{q_1, \dots, q_n\}^T$ the vector of generalized coordinates representing the position of the rigid bodies in the system;
- $\{u\}$ a vector of dependent coordinates;
- $r$ is the rank of the Jacobian matrix;
- $t$ time;
- $\gamma(q,\dot{q},t)$ is the accelerations vector;
- $\lambda$ is the array of Lagrange's multipliers;
- dots denote differentiation w.r.t. time.

## THE DIFFERENTIAL-ALGEBRAIC EQUATIONS SYSTEM

When using a redundant set of coordinates a number of constraint equations can be systematically established

$$\{\Psi(q,t)\} = \{\Psi_1(q,t) \dots \Psi_m(q,t)\}^T = \{0\} \qquad (1)$$

These constraints must be satisfied by the generalized coordinates throughout the simulation.

By differentiating (1) with respect to time, the velocity kinematic constraint equations are obtained

$$[\Psi_q]\{\dot{q}\} = -\{\Psi_t\}, \qquad (2)$$

where $[\Psi_q]$ is the Jacobian matrix of the system. Differentiating (2) again with respect to time leads to the accelerations kinematic constraint equations

$$[\Psi_q]\{\ddot{q}\} = \{\gamma\}, \qquad (3)$$

where

$$\{\gamma(q,\dot{q},t)\} = -([\Psi_q]\{\dot{q}\})_q\{\dot{q}\} - 2[\Psi_{qt}]\{\dot{q}\} - \{\Psi_{tt}\}. \qquad (4)$$

Systems of equations from (1) to (3) characterize the admissible motions. The d.o.f. of the mechanical system under analysis is [4]

$$F = n - r. \qquad (5)$$

The rank of the Jacobian matrix may vary during simulation.

The motion of the mechanical system under the influence of applied forces is governed by the following index 3 DAE system:

$$\begin{cases} [M]\{\ddot{q}\} + [\Psi_q]^T\{\lambda\} = \{F_e\} \\ \{\Psi(q,t)\} = \{0\} \end{cases} \qquad (6)$$

Thus the total number of equations is $n+m$ with $n+m$ unknowns, i.e., the $n$ generalized coordinates and $m$ Lagrange's multipliers related to the $m$ constraint equations.

It is well known that the task of obtaining a numerical solution from a DAE system is more difficult than that of solving an ODE system [5,6,10]. Analytical solutions of system (6) satisfy Eqs. (1) and (2), but this is not necessarily true for numerical solutions.

In this context several strategies are available for the numerical solution of the DAE of multibody dynamics. One of these rearranges the DAE system (6) into an ODE. In particular, in order to solve the system (6), without any orthogonalization, one could express (6) as follows

$$[K]\{\dot{y}\} = \{\phi(y)\} \qquad (7)$$

where

$$[K] = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad \{y\} = \begin{Bmatrix} q \\ \dot{q} \\ \ddot{q} \\ \lambda \end{Bmatrix} \qquad (8)$$

and

$$\{\phi(y)\} = \begin{Bmatrix} \dot{q}\} \\ \{\ddot{q}\} \\ \{[M]\{\ddot{q}\} + [\Psi_q]^T\{\lambda\} - \{F_e\}\} \\ \{\Psi\} \end{Bmatrix}. \qquad (9)$$

The form (7) is the one required by the Fortran subroutine *RADAU5* which is based on a fifth-order Runge–Kutta's method [6].

## THE ORTHOGONALIZATION METHOD

In order to transform (6) into an ODE system, where the unknown is a set $\{p\}$ of $F$ independent coordinates, the following algebraic manipulations should be been considered as reported in [7].

Let us append to the vector $\{\Psi\}$, the vector $\{\Phi\}$ of the equations which can be established between the coordinates $p$ and $q$

$$\{\Gamma(p,q)\}_{(m+F)\times 1} = \left\{ \begin{array}{c} \Psi(q) \\ \Phi(p,q) \end{array} \right\} \tag{10}$$

The time derivative of (10) leads to

$$[\Gamma_p]_{(m+F)\times F}\{\dot{p}\} + [\Gamma_q]_{(m+F)\times n}\{\dot{q}\} = \{0\} \tag{11}$$

where: $\left[\Gamma_p\right]_{(m+F)\times F} = \frac{\partial \Gamma}{\partial p}$ and $\left[\Gamma_q\right]_{(m+F)\times n} = \frac{\partial \Gamma}{\partial q}$ are Jacobian matrices.

From (11), after introducing the matrix

$$[V(p,q)]_{n\times F} = -\left[\Gamma_q\right]^{-1}_{n\times(m+F)}\left[\Gamma_p\right]_{(m+F)\times F} \tag{12}$$

one obtains

$$\{\dot{q}\} = [V(p,q)]\{\dot{p}\} \tag{13}$$

When there is no explicit dependence on time of the constraint equations $\{\Psi_t\} = \{0\}$ and combining the (2) with (13), the orthogonality condition is deduced

$$\left[\Psi_q\right][V(p,q)]\{\dot{p}\} = \{0\}. \tag{14}$$

Moreover when the Eq. 14 is differentiated with respect to time once again, the accelerations of generalized coordinates are expressed as function of the derivatives of $\{p\}$:

$$\{\ddot{q}\} = [V(p,q)]\{\ddot{p}\} + \left[\dot{V}(p,q)\right]\{\dot{p}\}. \tag{15}$$

Premultiplying both sides of the dynamic equation of system (6) by $[V]^T$ and taking into account (14) and (15), the vector of Lagrange's multipliers is eliminated from the differential equations of equilibrium and the following ODE system is obtained

$$\begin{aligned} &[V(p,q)]^T[M][V(p,q)]\{\ddot{p}\} = [V(p,q)]^T\{F\} \\ &\quad - [V(p,q)]^T[M]\left[\dot{V}(p,q)\right]\{\dot{p}\} \end{aligned} \tag{16}$$

Considering the term $\left[\dot{V}(p,q)\right]\{\dot{p}\}$, in order to obtain an expression ready to be implemented, the vector $\{q\}$ of generalized coordinates could be partitioned into two vectors as follow:

$$\{q\} = \left\{ \begin{array}{c} \{y\} \\ \{p\} \end{array} \right\} \tag{17}$$

where $\{y\}_{(n-F)\times 1}$ is the vector of dependent variables and $\{p\}_{F\times 1}$is the one of independent ones as already introduced. According to this, the kinematics expression of the acceleration $\left[\Psi_q\right]\{\ddot{q}\} = \{\gamma\}$ could be rearranged as:

$$\left[\Psi_y\right]_{m\times(n-F)}\{\ddot{y}\} + \left[\Psi_p\right]_{m\times F}\{\ddot{p}\} = \{\gamma\}. \tag{18}$$

From the expression (18) could be obtained the relationship between $\{\ddot{q}\}$and $\{\ddot{p}\}$as:

$$\begin{aligned} \{\ddot{q}\} &= \left\{ \begin{array}{c} \{\ddot{y}\} \\ \{\ddot{p}\} \end{array} \right\} = \left[ \begin{array}{c} -\left[\Psi_y\right]^{-1}\left[\Psi_p\right] \\ [I] \end{array} \right]\{\ddot{p}\} \\ &\quad + \left\{ \begin{array}{c} \left[\Psi_y\right]^{-1}\{\gamma\} \\ [0] \end{array} \right\} \end{aligned} \tag{19}$$

and comparing the (19) to (15) could be easily deduced that:

$$[V]_{n\times F} = \left[ \begin{array}{c} -\left[\Psi_y\right]_{(n-F)\times m}^{-1}\left[\Psi_p\right]_{m\times F} \\ [I]_{F\times F} \end{array} \right] \tag{20}$$

and

$$\left[\dot{V}(p,q)\right]\{\dot{p}\} = \left\{ \begin{array}{c} \left[\Psi_y\right]^{-1}\{\gamma\} \\ [0] \end{array} \right\}. \tag{21}$$

Now is also possible to define the matrix $[V]$ in the case of explicit dependence of the constraints with respect to time. So, in this case which is more common in the applications, the expression of the constraints vector is $\{\Psi(q,t)\} = \{0\}$ and the expression of velocities would change into:

$$\left[\Psi_q\right]\{\dot{q}\} = -\{\Psi_t\} \tag{22}$$

where the number of equations $m$ of this system would be less than the number of the elements $n$ of the vector of generalized coordinates $\{q\}$, then the Jacobian matrix would be a rectangular one.

Let us assume the correctness of the expression:

$$\{\dot{p}\} = [B]_{F\times n}\{\dot{q}\} \tag{23}$$

Because of singularities in the configuration of the mechanical system, the d.o.f. of the system may not be constant during the numerical integration proces. Moreover the matrix $[B]$ has the $F$ rows independent from the $m$ rows of the Jacobian matrix. As a consequence, combining the Eqs. 22 and 23 the following expression is obtained

$$\left[ \begin{array}{c} \left[\Psi_q\right] \\ [B] \end{array} \right]_{(m+F)\times n}\{\dot{q}\} = \left\{ \begin{array}{c} -\{\Psi_t\} \\ \{\dot{p}\} \end{array} \right\} \tag{24}$$

where the left hand side matrix is not singular. The solution of the previous system gives

$$\begin{aligned} \{\dot{q}\} &= \left[ \begin{array}{c} \left[\Psi_q\right] \\ [B] \end{array} \right]^{-1}_{n\times(m+F)} \left\{ \begin{array}{c} -\{\Psi_t\} \\ \{\dot{p}\} \end{array} \right\} \\ &= -[S]_{n\times m}\{\Psi_t\} + [V]\{\dot{p}\} \end{aligned} \tag{25}$$

where matrix $[S]$ is obtained from the first $m$ columns of matrix $\left[ \begin{array}{c} [\Psi_q] \\ [B] \end{array} \right]^{-1}$.

Eq. 25 contains, as a particular case, the expression (13). Solving the system (24) the following expression is obtained.

$$\begin{aligned}\begin{bmatrix}[\Psi_q]\\ [B]\end{bmatrix}\begin{bmatrix}[\Psi_q]\\ [B]\end{bmatrix}^{-1} &= \begin{bmatrix}[\Psi_q]\\ [B]\end{bmatrix}[[S][V]]\\ &= \begin{bmatrix}[\Psi_q][S] & [\Psi_q][V]\\ [B][S] & [B][V]\end{bmatrix} = \begin{bmatrix}I & 0\\ 0 & I\end{bmatrix}\end{aligned} \tag{26}$$

which confirms the property of orthogonality also in the case of reonomic constraints:

$$[\Psi_q][V] = [0] \tag{27}$$

and introduces the new one

$$[B][V] = [I]. \tag{28}$$

Deriving with respect to time Eq. 24 the following expression would be obtained:

$$\begin{bmatrix}[\Psi_q]\\ [B]\end{bmatrix}\{\ddot{q}\} = \begin{Bmatrix}\{\gamma\}\\ \{\ddot{p}\}\end{Bmatrix} \tag{29}$$

which carries out the relationship between $\{\ddot{p}\}$and $\{\ddot{q}\}$as

$$\{\ddot{q}\} = [S]\{\gamma\} + [V]\{\ddot{p}\}. \tag{30}$$

The term $[S]\{\gamma\}$ is obtained from Eq. 30, assuming $\{\ddot{p}\} = 0$, as follow

$$[S]\{\gamma\} = \begin{bmatrix}[\Psi_q]\\ [B]\end{bmatrix}^{-1}\begin{Bmatrix}\{\gamma\}\\ \{0\}\end{Bmatrix}. \tag{31}$$

## THE DEDUCTION OF MATRIX [V]

In this section two methods to deduce matrix $[V]$ previously introduced will be presented. Both of them could be easily implemented. It must be understood that the matrix $[V]$ is not unique.

### QR Decomposition

According to this method, the Jacobian matrix $[\psi_q]^T$ is first rearranged as follows

$$[\Psi_q]^T = [\,[Q_1]_{n\times m}\ [Q_2]_{n\times F}\,]\begin{bmatrix}[R_1]_{m\times m}\\ [0]_{F\times m}\end{bmatrix} = [Q_1][R_1] \tag{32}$$

where $[Q_1]$ and $[Q_2]$ simultaneously satisfy the conditions

$$\begin{aligned}[Q_2]^T[Q_1] &= [0]\\ [Q_2]^T[Q_2] &= [I]\end{aligned} \tag{33}$$

The first one is used to eliminate the Lagrange's multipliers in the equations of motion. In fact, considering Eq. 32 and the first one of (33), the following one is obtained:

$$[Q_2]^T[\Psi_q]^T = [Q_2]^T[Q_1][R_1] = [0] \tag{34}$$

Comparing (34) and the second one of (33), together with (27) and (28) respectively, matrix $[V]$ is deduced as:

$$\begin{aligned}[V] &= [Q_2]\\ [B] &= [Q_2]^T\end{aligned} \tag{35}$$

Hence, the equations of motion could be rearranged as follow:

$$[Q_2]^T[M][Q_2]\{\ddot{p}\} = [Q_2]^T\{F_e\} - [Q_2]^T[M][S]\{\gamma\} \tag{36}$$

where

$$[S]\{\gamma\} = \begin{bmatrix}[\Psi_q]\\ [Q_2]^T\end{bmatrix}^{-1}\begin{Bmatrix}\{\gamma\}\\ \{0\}\end{Bmatrix}. \tag{37}$$

### Singular Value Decomposition

Matrix $[V]$ is now deduced by means of the Singular Value Decomposition (SVD) algorithm. Therefore, matrix $[\Psi_q]^T$has been rearranged as follows

$$\begin{aligned}[\Psi_q]^T &= [\,[W_d]_{n\times r}\ [W_i]_{n\times(n-r)}\,]\begin{bmatrix}[\Lambda_1]_{r\times r}\\ [0]_{(n-r)\times r}\end{bmatrix}[U]^T_{r\times r}\\ &= [W_d][\Lambda_1][U]^T.\end{aligned} \tag{38}$$

The columns of the matrix $[[W_d][W_i]]$must be orthogonal, i.e., $[W_i]^T[W_d] = [0]$. Hence, combining this with (38) the following expression is obtained

$$[W_i]^T[\Psi_q]^T = [W_i]^T[W_d][\Lambda_1][U]^T = [0]. \tag{39}$$

Matrix $[V]$ is then defined as

$$[V] = [W_i] \tag{40}$$

and, as a consequence, the following equalities are established

$$\{\dot{q}\} = [W_i]\{\dot{p}\} \tag{41}$$

and

$$[S]\{\gamma\} = \begin{bmatrix}[\Psi_q]\\ [W_i]^T\end{bmatrix}^{-1}\begin{Bmatrix}\{\gamma\}\\ \{0\}\end{Bmatrix}. \tag{42}$$

The final expression of the equations of motion is

$$[W_i]^T[M][W_i]\{\ddot{p}\} = [W_i]^T\{F\} - [W_i]^T[M][S]\{\gamma\}. \tag{43}$$

## COORDINATE PARTITIONING METHOD

This method is a state space method which use a subset of the generalized coordinates to locally parameterize the constraint manifold [1−3,8,9]. As previously seen the generalized coordinates are partitioned into a vector $\{p\}$ of $F$ independent coordinates and the remaining $m$ generalized coordinates are rearranged into vector $\{u\}$. Note that $F+m=n$ and that the partitioning is done such that the sub-Jacobian of the constraint equations with respect to $\{u\}$ is nonsingular, i.e.,

$$\det(\Psi_u(q,t)) \neq 0. \tag{44}$$

This partitioning strategy is only feasible when the constraint equations are independent, that is the Jacobian $[\Psi_q]$ has full row rank.

According to this partitioning scheme, the DAE system (6) are rewritten in the form

$$\begin{cases} [M^{pp}]\{\ddot{p}\} + [M^{pu}]\{\ddot{u}\} + \left[\Psi_p\right]^T\{\lambda\} = \{Fe^p\} \\ [M^{up}]\{\ddot{p}\} + [M^{uu}]\{\ddot{u}\} + [\Psi_u]^T\{\lambda\} = \{Fe^u\} \\ \{\Psi(u,p,t)\} = \{0\} \end{cases} \tag{45}$$

The velocity and acceleration kinematic constraint equations can also be rearranged into

$$[\Psi_u]_{m\times(n-F)}\{\dot{u}\} + \left[\Psi_p\right]_{m\times F}\{\dot{p}\} = -\{\Psi_t\} \tag{46}$$

and

$$[\Psi_u]\{\ddot{u}\} + \left[\Psi_p\right]\{\ddot{p}\} = \{\gamma\} \tag{47}$$

respectively.

According to the condition of Eq. 44 and the implicit function theorem, $\{u\}$ can be locally represented as a function of $\{p\}, \{\mathrm{u}\} = h(\{p\})$, when $h(\{p\})$ has as many continuous derivatives as the constraint function $\Psi(q,t)$.

Using information provided by (46) and $\{u\} = h(\{p\})$ the DAE system (45) can be reduced to a State Space Ordinary Differential Equations (SSODE) system. Thus, since $[\Psi_u]$ in (46) is nonsingular, $\{\dot{u}\}$ can be determined as a function of $\{\dot{p}\}$ and $\{p\}$ eliminating the explicit dependence on $\{u\}$ by means of the relationship between $\{p\}$ and $\{u\}$. Next, Eq. 47 uniquely determines $\{\ddot{u}\}$ as a function of $\{p\}$, $\{\dot{p}\}$, and $\{\ddot{p}\}$ where results from Eq. 46 and the relationship between $\{p\}$ and $\{u\}$ are substituted. In the same way, using previous results, $\{\lambda\}$ can be uniquely determined as a function of $\{p\}$, $\{\dot{p}\}$, and $\{\ddot{p}\}$. Finally the SSODE in the independent generalized coordinates $\{p\}$ is obtained,

$$\left[\widehat{M}(p)\right]\{\ddot{p}\} = \left\{\widehat{F}(t,p,\dot{p})\right\}, \tag{48}$$

where

$$\left[\widehat{M}\right] = [M^{pp}] - [M^{pu}][\Psi_u]^{-1}\left[\Psi_p\right] - \left[\Psi_p\right]^T\left([\Psi_u]^{-1}\right)^T \left([M^{up}] - [M^{uu}][\Psi_u]^{-1}\left[\Psi_p\right]\right) \tag{49}$$

$$\left\{\widehat{F}\right\} = \{Fe^p\} - [M^{pu}][\Psi_u]^{-1}\{\gamma\} - \left[\Psi_p\right]^T\left([\Psi_u]^{-1}\right)^T \left(\{Fe^u\} - [M^{uu}][\Psi_u]^{-1}\{\gamma\}\right) \tag{50}$$

## AN EXAMPLE OF THE DECOMPOSITION METHODS EXPLAINED

For a better understanding of the theory explained a simple example is discussed. With reference to the nomenclature of Figure 1, the constraint equations, the Jacobian matrix and the vector of accelerations are respectively:

$$\{\Psi\} = \begin{Bmatrix} q_1 - L\cos q_3 \\ q_2 - L\sin q_3 \end{Bmatrix}, \quad \left[\Psi_q\right] = \begin{bmatrix} 1 & 0 & L\sin q_3 \\ 0 & 1 & -L\cos q_3 \end{bmatrix}, \tag{51}$$

$$\{\gamma\} = \{\, -L\dot{q}_3\cos q_3 \quad -L\dot{q}_3\sin q_3 \,\}^T$$

QR decomposition. Matrix $[V]$ orthogonal to matrix $[\psi_q]^T$ is deduced as follow

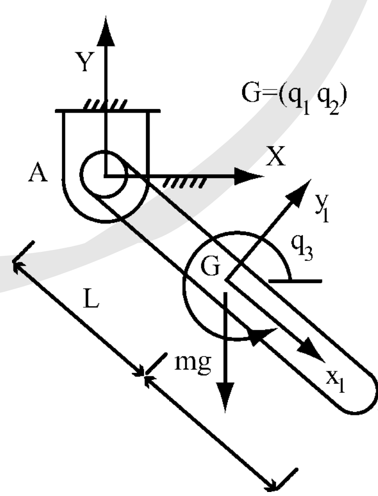


**Figure 1** Simple pendulum.

$$[\Psi_q]^T = [\,[Q_1]\ [Q_2]\,]\begin{bmatrix}[R_1]\\ [0]\end{bmatrix} \tag{52}$$

where

$$[R_1] = \begin{bmatrix}\sqrt{1-L^2\sin^2 q_3} & -\frac{L^2\sin q_3\cos q_3}{\sqrt{1-L^2\sin^2 q_3}}\\ 0 & \sqrt{\frac{1+L^2}{1-L^2\sin^2 q_3}}\end{bmatrix} \tag{53}$$

$$[Q_2] = [\,-L\sin q_3 \quad L\cos q_3 \ \ 1\,]^T$$

Therefore, since $[Q2]=[V]$, the equation $\{\dot{q}\} = [V]\{\dot{p}\}$ becomes

$$\begin{Bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{Bmatrix} = \begin{bmatrix}-L\sin q_3\\ L\cos q_3\\ 1\end{bmatrix}\{\dot{p}\}. \tag{54}$$

Moreover, from Eq. 37 follows

$$[S]\{\gamma\} = \begin{bmatrix}[\Psi_q]\\ [Q_2]^T\end{bmatrix}^{-1}\begin{Bmatrix}\{\gamma\}\\ \{0\}\end{Bmatrix} = \begin{bmatrix}1 & 0 & L\sin q_3\\ 0 & 1 & -L\cos q_3\\ -L\sin q_3 & L\cos q_3 & 1\end{bmatrix}^{-1}\begin{Bmatrix}-L\dot{q}_3\cos q_3\\ -L\dot{q}_3\sin q_3\\ 0\end{Bmatrix} = \begin{Bmatrix}-L\dot{q}_3^2\cos q_3\\ -L\dot{q}_3^2\sin q_3\\ 0\end{Bmatrix}. \tag{55}$$

In conclusion, denoted with

$$[M] = \begin{bmatrix}m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I_g\end{bmatrix},$$

the mass matrix, $\{F_e\} = \{\,0\ -mg\ 0\,\}^T$, the vector of external forces, the system (36) is

$$(mL^2 + I_g)\ddot{p} + mgL\cos q_3 = 0 \tag{56}$$

where $\ddot{p}$ is the only unknown.

## Singular Value Decomposition (SVD)

Using this approach one can only proceed numerically (computer algebra systems apply the SVD algorithm only on matrices expressed in numerical form). Let $L = 1$m and, as initial position, impose $q_3 = \pi/4$. Thus Jacobian and acceleration vector are

$$[\Psi_q] = \begin{bmatrix}1\ 0 & \frac{\sqrt{2}}{2}\\ 0\ 1 & -\frac{\sqrt{2}}{2}\end{bmatrix} \text{ and } \{\gamma\} = \left\{-\frac{\sqrt{2}}{2}\dot{q}_3^2 - \frac{\sqrt{2}}{2}\dot{q}_3^2\right\}^T. \tag{57}$$

According to the SVD algorithm the $[\psi_q]^T$ matrix can be decomposed into three matrices, i.e.,

$$[\Psi_q]^T = [W][\Lambda][U]^T \tag{58}$$

where, for the case under analysis,

$$[W] = \begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{2}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2}\\ -\frac{\sqrt{2}}{2} & 0, & \frac{\sqrt{2}}{2}\end{bmatrix},\ [\Lambda] = \begin{bmatrix}\sqrt{2} & 0\\ 0 & 1\\ 0 & 0\end{bmatrix},$$
$$[U] = \begin{bmatrix}-\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\end{bmatrix}. \tag{59}$$

Comparing the matrices in the expression (59) with the ones in (38) it is easy to deduce that

$$[W_i] = \begin{bmatrix}-\frac{1}{2}\\ \frac{1}{2}\\ \frac{\sqrt{2}}{2}\end{bmatrix} = \begin{bmatrix}-\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\\ 1\end{bmatrix} \tag{60}$$

and

$$[S]\{\gamma\} = \begin{bmatrix}[\Psi_q]\\ [W_i]^T\end{bmatrix}^{-1}\begin{Bmatrix}\{\gamma\}\\ \{0\}\end{Bmatrix} = \begin{Bmatrix}-\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}\\ 0\end{Bmatrix}\dot{q}_3^2. \tag{61}$$

In conclusion, using Eq. 43, the following differential equation is obtained

$$(m + I_g)\ddot{p} + \frac{\sqrt{2}}{2}gm = 0 \tag{62}$$

which is the same of the previous case considering the numerical data assigned.

Coordinate partitioning method. According to this method the generalized coordinate are splitted into $\{p\}$ and $\{u\}$ vectors as follows,

$$\{u\} = \{q_1, q_2\} \text{ and } \{p\} = \{q_3\}.$$

The Jacobian matrices obtained by differentiation of the constraint equations with respect to $\{u\}$ and $\{p\}$ respectively are

$$[\Psi_u] = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad \text{and} \quad [\Psi_p] = \begin{bmatrix}L\sin q_3\\ -L\cos q_3\end{bmatrix}.$$

The global mass matrix is partitioned into the following sub-matrices

$$[M^{pp}] = [I_g],\ [M^{pu}] = [0 \quad 0],\ [M^{up}] = \begin{bmatrix}0\\ 0\end{bmatrix},$$

$$\text{and } [M^{uu}] = \begin{bmatrix}m & 0\\ 0 & m\end{bmatrix}.$$

**Table 1** Joint Definition Frame

| CON | B1 | B2 | T | $P_{1x}$ | $P_{1y}$ | $P_{1z}$ | $Q_{1x}$ | $Q_{1y}$ | $Q_{1z}$ | $R_{1x}$ | $R_{1y}$ | $R_{1z}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | −1 | 0 | 0 |
| | | | | $P_{2x}$ | $P_{2y}$ | $P_{2z}$ | $Q_{2x}$ | $Q_{2y}$ | $Q_{2z}$ | $R_{2x}$ | $R_{2y}$ | $R_{2z}$ |
| | | | | −0.5 | 0 | 0 | −0.5 | 0 | 1 | −1.5 | 0 | 0 |

**Table 2** Rotational Spring Damper Element

| NSPDAMP | 1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | B1 | B2 | K | $L_0$ | C | A | $P_{1x}$ | $P_{1y}$ | $P_{1z}$ | $P_{2x}$ | $P_{2y}$ | $P_{2z}$ | T |
| SP1 | 0 | 1 | 32.66 | 1.57 | 4.584 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |

**Table 3** Inertial Properties

| B | MAS | $I_{xx}$ | $I_{xy}$ | $I_{xz}$ | $I_{yx}$ | $I_{yy}$ | $I_{yz}$ | $I_{zx}$ | $I_{zy}$ | $I_{zz}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 0 | 0 | 0 | 0 | 0.834 | 0 | 0 | 0 | 0.834 |

**Table 4** External Forces

| NFOR | 1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| FOR | BO | $F_x$ | $F_y$ | $F_z$ | $T_x$ | $T_y$ | $T_z$ | $P_x$ | $P_y$ | $P_z$ | Rif |
| F1 | 1 | 0 | 0 | −98.10 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The vector of generalized forces is partitioned as well,

$$\{Fe^u\} = \{\, 0 \; -mg \,\}^T \text{ and } \{Fe^p\} = \{0\},$$

the expression of vector $\{\gamma\}$is not changed instead.

According to Eq. 49 the terms of the mass matrix $[\hat{M}]$ are

$$[\Psi_p]^T\left([\Psi_u]^{-1}\right)^T\left([M^{up}]-[M^{uu}][\Psi_u]^{-1}[\Psi_p]\right) = [-mL^2], \tag{63}$$

$$[M^{pu}][\Psi_u]^{-1}[\Psi_p] = [0], \tag{64}$$

$$[M^{pp}] = [I_g] \tag{65}$$

thus

$$[\hat{M}] = [I_g + mL^2]; \tag{66}$$

**Table 5** Initial Condition

| DOF | 1 | | |
|---|---|---|---|
| BO | Q | Pos | Vel |
| 1 | 4 | 0.97 | 0 |

**Table 6** CPU Time Consumption

| CODE | CPU time (s) |
|---|---|
| NumDyn3D | 20 |
| NumDyn3D with SVD | 16 |
| Working Model 3D | 30 |

the terms in the Eq. 50 concerning vector $\{\hat{F}\}$ are

$$[\Psi_p]^T\left([\Psi_u]^{-1}\right)^T\left(\{Fe^u\} - [M^{uu}][\Psi_u]^{-1}\{\gamma\}\right) = \{mLg \cos q_3\} \tag{67}$$

$$[M^{pu}][\Psi_u]^{-1}\{\gamma\} = \{0\}, \tag{68}$$

$$\{Fe^p\} = \{0\} \tag{69}$$

thus

$$\{\hat{F}\} = \{-mLg \cos q_3\}. \tag{70}$$

Finally the SSODE system according to (48) is

$$(I_g + mL^2)\ddot{p} = -mLg \cos q_3, \tag{71}$$

which is the same of the others reported previously.

## The Code NumDyn3D

A brief description of the main features of the code is presented in this section. NumDyn3D, the acronym of Numerical Dynamics in 3D, is a multibody dynamics

**Table 7** Routines

| Routines | QR | SVD |
|---|---|---|
| *IMSL* | LQRRR/DLQRRR | LSVRR/DLSVRR |
| *MATLAB* | Qr | SVD |
| *MAPLE* | Qrdecomp | Svd |
| *LAPACK* | SGEQPF | SGESVD |

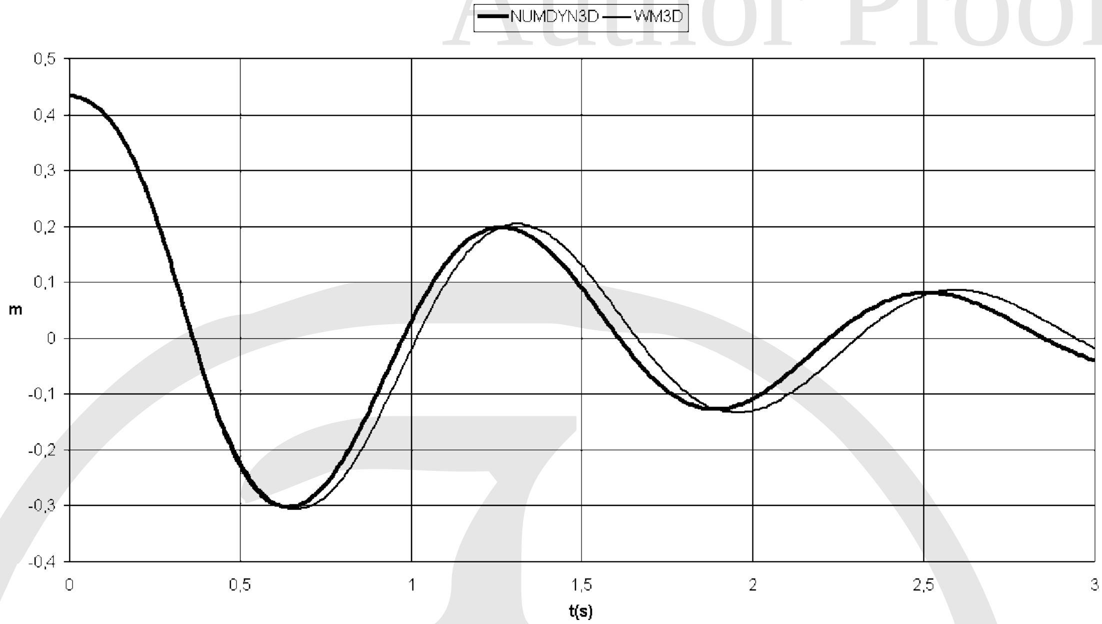


**Figure 2** Abscissa of center of mass in the *x* direction.

code developed by L. Vita at University of Rome Tor Vergata for didactic and research purposes. Entirely written in Fortran90, the software is suitable for the dynamic simulation of a large class of mechanical system. The use of the subroutine for the numerical integration *RADAU5*[6] ensures the robustness of the code and numerical accuracy of the results. The NumDyn3D code makes use of eleven text files to obtain the information needed to completely describe the mechanical system under investigation. The code generates, as output of the analysis process, one report file where all the input features of the system are summarized, and three files for each body of the system. Each file, in ASCII format, contains the positions, velocities and accelerations of the body at each integration step.

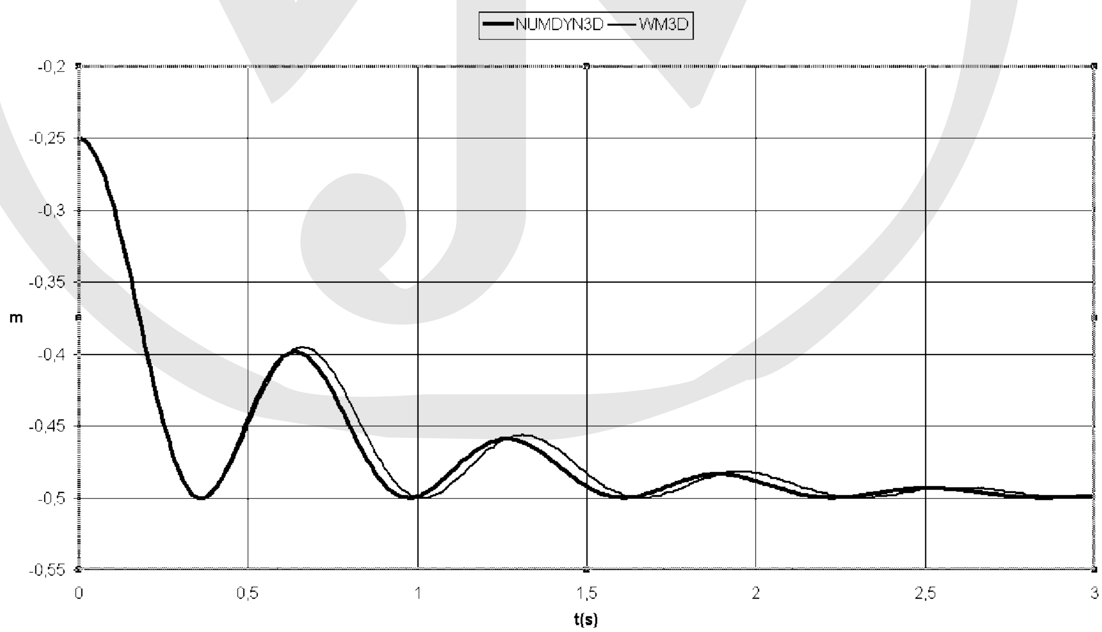


**Figure 3** Ordinate of center of mass in the *y* direction.

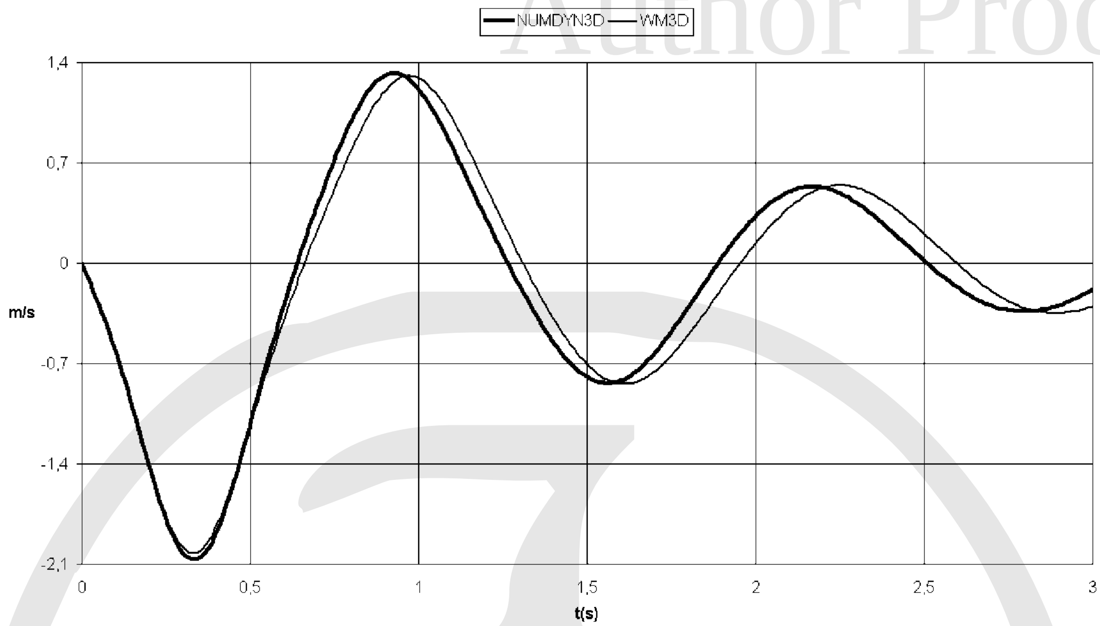


**Figure 4** Center of mass velocity ($x$ direction).

## THE BENCHMARK

In this section the results of the analysis of a simple pendulum (Fig. 1) obtained using NumDyn3D, with and without the othogonalization of the Jacobian matrix are compared with the results obtained by means of the commercial software Working Model 3D.

The analysis refers to three seconds of simulation divided into 3000 steps; there is only one revolute joint between the pendulum and the ground which is described using the appropriate text file as reported in Table 1 where the points of the joint definition frame for each body (in local coordinates) are defined.

A rotational spring damper element is also included in the model with the features reported in Table 2;

Whereas the inertial features are summarized in Table 3.

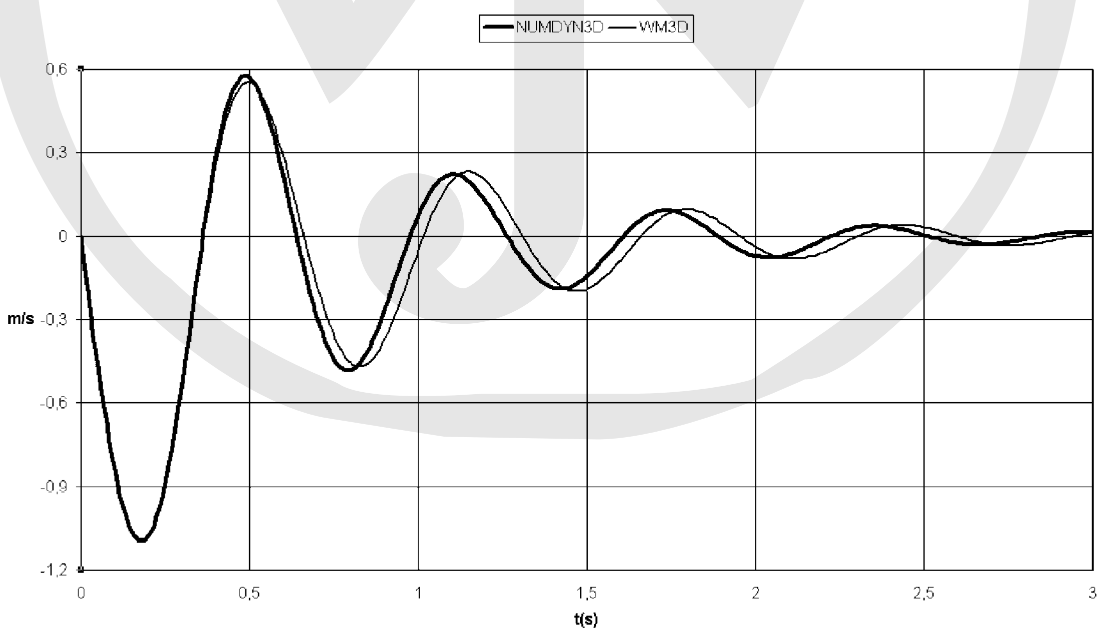


**Figure 5** Center of mass velocity ($y$ direction).

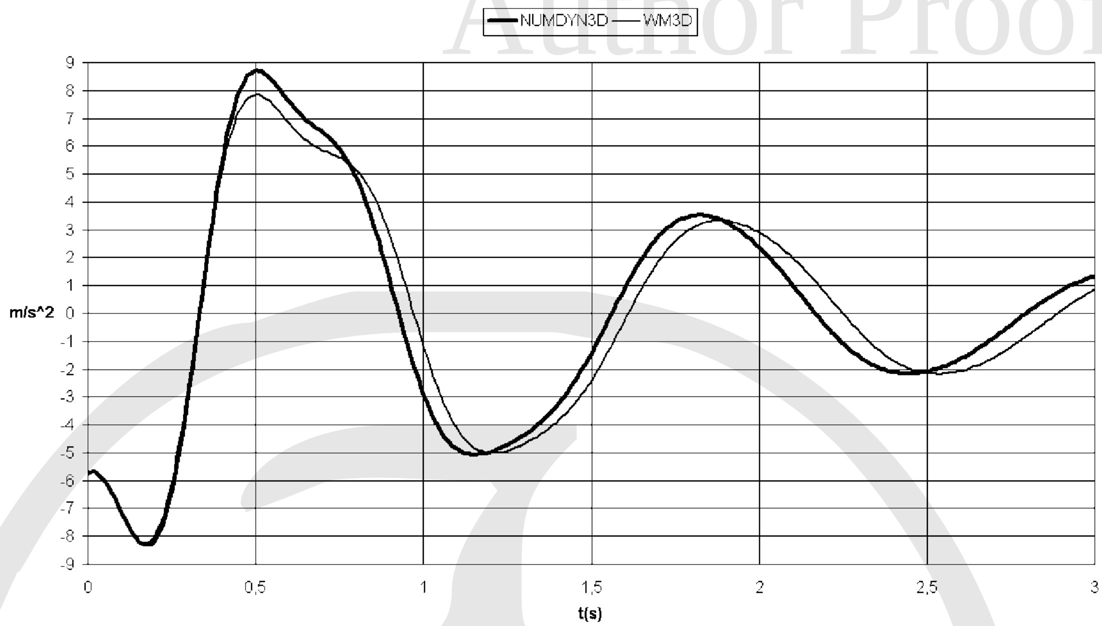


**Figure 6** Center of mass acceleration ($x$ direction).

The only external force acting on the system is the gravitation field one as shown in Table 4.

Considering that the system has only one d.o.f., just one parameter is required as initial condition. In this case the first Euler's parameter of the body as in Table 5.

The output obtained from the code is compared to the one obtained with Working Model 3D as shown in Figures 2−7.

Considering Figures 2−7 the differences between NumDyn3D and WM3D may not seem relevant. However, Working Model 3D integrates the system of equations by considering constraints on accelerations (DAE system of index 1). NumDyn3D chooses the more demanding task of integrating DAE system with positions constraints. This allows a very tight control of numerical accuracy. In fact with NumDyn3D,

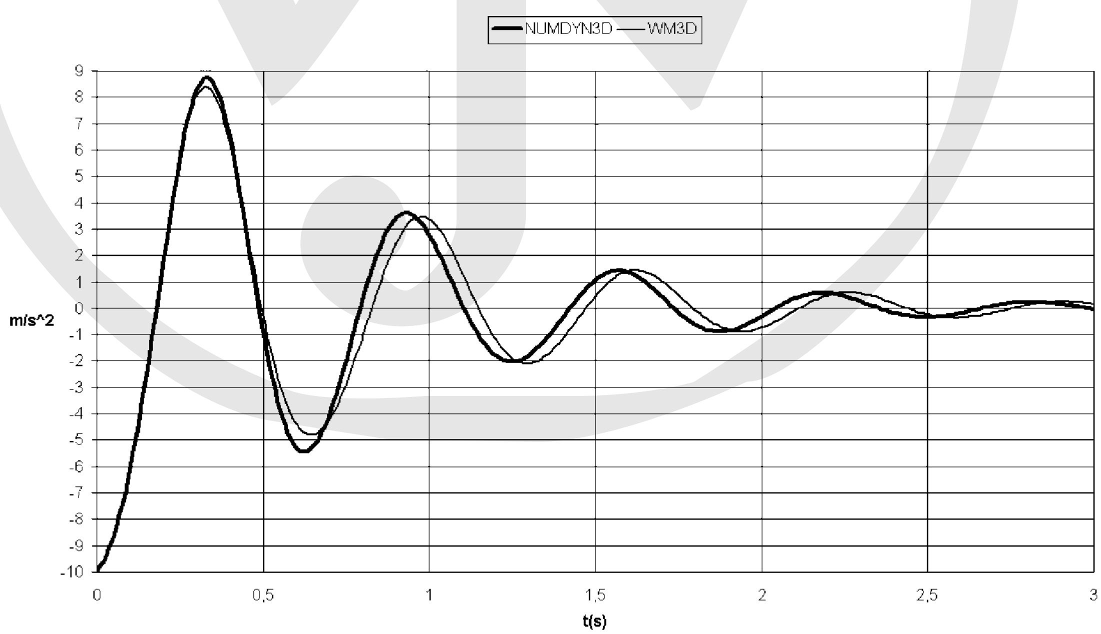


**Figure 7** Center of mass acceleration ($y$ direction).

the violation of constraints in this example is never greater than 2.6 $10^{-17}$ m.

Finally Table 6 shows the differences considering CPU time consumption. The orthogonalized version has a reduction of CPU time consumption about the 20% and a maximum error in the solution of the constraints equal to 5.0 $10^{-20}$. These performance improvements are due to the simplification of the system to be integrated obtained by the orthogonalization.

The reported differences may not be maintained for other examples.

In Table 7 are reported the routines useful for the implementation of one of the methods proposed using different software environments.

## CONCLUSIONS

Some strategies of numerical integration of DAE system are discussed and implemented for the simple case of the one d.o.f. pendulum. This paper reports all the steps involved in each method and it is believed to be useful for the teaching of this topic in multibody dynamics courses.

## REFERENCES

[1] E. J. Haug, Computer-aided kinematics and dynamics of mechanical systems, vol. I, Allyn and Bacon, 1989, pp 48–104.

[2] P. Nikravesh, Spatial kinematic and dynamic analysis with Euler parameters. In: E. J. Haug, editor. Computer aided analysis and optimization of mechanical system dynamics, NATO ASI Series, Springer Verlag, 1984, pp 261–281.

[3] A. A. Shabana, Computational dynamics, John Wiley & Sons, New York, 1994.

[4] E. T. Whittaker, A treatise on the analytical dynamics of particles and rigid bodies, Cambridge University Press, Cambridge (UK), 1937.

[5] K. E. Brennan, S. L. Campbell, L. R. Petzold, Numerical solution of initial-value problems in differential-algebraic equations, society for industrial and applied mathematics, 1996.

[6] E. Hairer, G. Wanner, Solving ordinary and differential equations II, stiff and differential-algebraic problems, Springer Verlag, 1991.

[7] E. Pennestrì, Dinamica tecnica e computazionale, vol. II, Ed. Ambrosiana, 2002, pp 377–412.

[8] D. Negrut, E. J. Haug, H. C. German, An implicit Runge–Kutta method for integration of differential algebraic equations of multibody dynamics, multibody system dynamics, Kluwer Academic Publisher, 2003, pp 121–142.

[9] P. J. Rabier, W. C. Rheinboldt, Nonholonomic motion of a rigid mechanical systems from a DAE viewpoint, Society for Industrial and Applied Mathematics, 2000.

[10] E. Pennestrì, S. Mingardi, A short review of numerical methods and software tools for solving differential-algebraic equations, Proceedings of Sixth Applied Mechanisms and Robotics Conference, December 12–15 1999, Cincinnati, OH, Paper No. AMR99-053.

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