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A DUAL NUMBER APPROACH TO THE KINEMATIC ANALYSIS OF SPATIAL LINKAGES WITH DIMENSIONAL AND GEOMETRIC TOLERANCES

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ABSTRACT

Design robustness is somewhat connected to tolerances. In fact, the lower is the sensitivity of the kinematic function to the deviations of manufacturing process, the higher is the robustness of the design.

In this investigation is described a tolerance analysis method based on dual vectors kinematic modeling of spatial linkages and on Monte Carlo simulation of the random variables. In the present analysis the hypothesis of rigid bodies is valid and only kinematic variables are considered in output. The method is applied to a Cardan joint modelled as an RCCC linkage with main dimensions considered as stochastic variables with Gaussian distribution.

Dual vectors are well known in kinematic analysis and synthesis of spatial mechanisms. When compared with traditional vectorial methods, dual vectors show an enhanced capability to model misalignments among kinematic pairs axes. Although this is not the first time that dual vectors are used for the kinematic and dynamic analysis of spatial mechanisms with manufacturing errors, the present use of dual vectors to model joint clearances seems somewhat novel.

NOMENCLATURE

$\left[{}^{j}_{i}\widehat{A}\right]$	dual transform matrix from reference system $O_i - x_i y_i z_i$ on link i to $O_j - x_j y_j z_j$, on link j;
I	identity matrix;
H	nominal length of the shaft axis;
r	nominal shaft radius;
R	nominal bearing radius;
R_i	radius of the ideal spherical link;
ε	dual unit, $\varepsilon^2 = 0$;
τ_{ij}	angle between bearing axis and journal axis in presence of clearances;
$\widehat{.}$	denote dual quantities;
$\{.\}$ and $[.]$	denote vectors and matrices, respectively;
Subscript i_k	denotes an item expressed in the k^{th} joint Cartesian frame of link i;

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INTRODUCTION

It is well known the impact on manufacturing costs and product quality of placing tight tolerances on CAD drawings. On the other hand, loose tolerances involve the risk of poor performances. Thus the designer has to compromise between these two extreme alternatives. The number of rejects per number of assemblies is the *assembly yield* or *assembly acceptance fraction*. This is usually both a design requirement and a quality index during simulation.

At the practical level the engineer is faced with two different types of tasks:

> **Tolerance analysis**. This task consist in the estimate of the assembly yield through analytical modeling of mechanical assemblies taking into account the variations introduced by the components at each assembly configuration.
> **Tolerance allocation**. This is the assignment and distribution of component tolerances in order to meet assembly yield requirements within the appropriate cost and feasibility constraints [2, 3].

In general, tolerance analysis involves the building of linear or nonlinear models of the mechanical assembly. By running the models one can monitor the effects of design parameter variations on the fluctuations of product performances.

Design robustness is somewhat connected to tolerances. In fact, the lower is the sensitivity of the kinematic function to the deviations of manufacturing process, the higher is the robustness of the design.

In this investigation a tolerance analysis method based on dual vectors kinematic modeling of the spatial linkage and on Monte Carlo simulation of the random variables. In the present analysis the hypothesis of rigid bodies is valid and only kinematic variables are considered in output. The method is applied to Cardan joint with manufacturing tolerances and skewed kinematic pairs axes. For this reason the Cardan joint is modeled as an RCCC spatial linkage with main dimensions considered as stochastic variables with Gaussian distribution. The methodology has been tested on this case which we do consider significant from the industrial point of view. However, the methodology can be applied to spatial linkages with revolute and cylindrical pairs.

Dual vectors are well known in kinematic analysis and synthesis of spatial mechanisms *e.g.* [7, 8]. When compared with traditional vectorial methods, dual vectors show an enhanced capability to model misalignments among kinematic pairs axes. Although this is not the first time that dual vectors are used for the kinematic and dynamic analysis of spatial mechanisms with manufacturing errors *e.g.* [13, 14], the present use of dual vectors to model joint clearances seems somewhat novel.

OUTLINE OF TOLERANCE ANALYSIS METHODOLOGIES

There is a high number of scientific contributions on the problem and this review cannot be exhaustive.
The theoretical tools used in previous work on kinematic tolerance analysis falls into three main categories [5]:

> **Standard worst-case analysis.** The maximum and minimum values associated with all components in an assembly are used in order to compute the assembly's range of variation in kinematic performances [1].
> **Monte Carlo simulation.** The designer can perform statistical analyses of the uncertainty characteristic of tolerance analysis problems. A Monte Carlo analysis involves the generation of one set of n random numbers for each stochastic variable in the analysis equations. These are solved n times by using each random number in the set. The values of the responses are analysed using histograms and sample statistics.
> **Statistical.** Statistical distributions can be used to predict the yield of an assembly. Different methodologies have been developed (Root sum squares, Mean shift, Six sigma, etc.)

[4] describe a procedure for tolerance analysis of spatial mechanisms based on the use of Denavit-Hartenberg parameters. The mechanisms kinematics is described by means of vector loop equations. Tolerance sensitivity matrices of the assembly are computed.
Tolerance analysis is based on imprecise and often not complete data descriptions. The designer is requested to foresee output quantities from imprecise input parameters and rank different design alternatives. Computational tools, like the one discussed in this paper, may assist the designer to foresee the product performances. In such situations statistics appears as the most appropriate theoretical tool for representing and manipulating the uncertainty in real design problems.

THEORETICAL BASES

Most methods of kinematic analysis based on dual vectors make use of the dual Denavit-Hartenberg parameters. In some cases (for instance in spherical linkages) these are not uniquely defined and have many limitations. We adapted to dual vectors the method presented in the textbooks of [9] and [10].
In particular, a Cartesian reference frame $O_i - x_i y_i z_i$ $(i = 1, 2, \ldots, n)$ is attached to the i^{th} link. Other Cartesian reference frames $P_{i_k} - x_{i_k} y_{i_k} z_{i_k}$ $(k = 1, 2, \ldots, m)$ are attached to each kinematic element of the link. The relative position of the two Cartesian frames is prescribed in terms of the coordinates of three points $P_{i_k} \equiv (P_x, P_y, P_z)$, $Q_{i_k} \equiv (Q_x, Q_y, Q_z)$ ed $R_{i_k} \equiv (R_x, R_y, R_z)$. With reference to Figure 1:

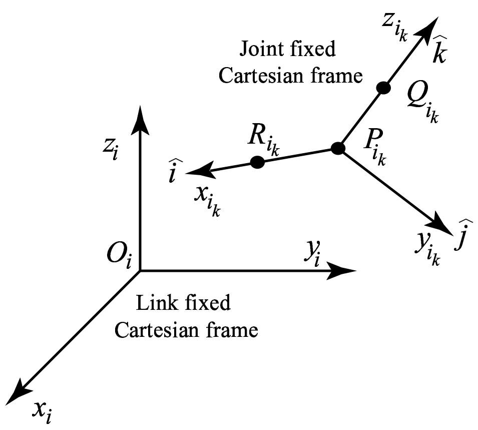

Figure 1. Nomenclature

- the point P_{i_k} coincides with the origin O_{i_k};
- the point R_{i_k} is on coordinate axis x_{i_k} at unit distance from P_{i_k};
- the point Q_{i_k} is on coordinate axis z_{i_k} at unit distance from P_{i_k}.

Therefore, the components of axes versors are

$$\{i\} = \left\{ R_x - P_x \; R_y - P_y \; R_z - P_z \right\}^T , \tag{1a}$$

$$\{k\} = \left\{ Q_x - P_x \; Q_y - P_y \; Q_z - P_z \right\}^T , \tag{1b}$$

$$\{j\} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix} \begin{Bmatrix} i_x \\ i_y \\ i_z \end{Bmatrix} . \tag{1c}$$

These versors, when considered as line versors, are represented by the following dual versors

$$\left\{\widehat{i}\right\} = \{i\} + \varepsilon \left[\widetilde{OP}\right] \{i\} , \tag{2a}$$

$$\left\{\widehat{k}\right\} = \{k\} + \varepsilon \left[\widetilde{OP}\right] \{k\} , \tag{2b}$$

$$\left\{\widehat{j}\right\} = \begin{bmatrix} 0 & -\widehat{k}_z & \widehat{k}_y \\ \widehat{k}_z & 0 & -\widehat{k}_x \\ -\widehat{k}_y & \widehat{k}_x & 0 \end{bmatrix} \begin{Bmatrix} \widehat{i}_x \\ \widehat{i}_y \\ \widehat{i}_z \end{Bmatrix} , \tag{2c}$$

where

$$\left[\widetilde{OP}\right] = \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix} . \tag{3}$$

The transform matrix of line vectors from $O_i - x_i y_i z_i$ to $O_{i_k} - x_{i_k} y_{i_k} z_{i_k}$ has the form

$$\left[{}^{i_k}_{i}\widehat{A}\right] = \begin{bmatrix} \widehat{i}_x & \widehat{j}_x & \widehat{k}_x \\ \widehat{i}_y & \widehat{j}_y & \widehat{k}_y \\ \widehat{i}_z & \widehat{j}_z & \widehat{k}_z \end{bmatrix} . \tag{4}$$

For each loop in the spatial linkage a matrix closure equation can be written.
For instance, for an RCCC linkage, assuming a consecutive numbering of the links, such equation takes the form (with reference to Fig. 6)

$$\prod_{i=1}^{4} \left[{}^{i_1}_{i}\widehat{A}\right] \left[{}^{i_2}_{i_1}\widehat{A}\right] \left[{}^{(i+1)_1}_{i_2}\widehat{A}\right] \left[{}^{(i+1)}_{(i+1)_1}\widehat{A}\right] = [I] , \tag{5}$$

where $i+1 = 1$ when $i = 4$.

THE MODELING OF A CYLINDRICAL JOINT

In this section we will describe the kinematic modeling of a cylindrical joint. First the ideal case will be described and then the presence of clearances is introduced.
The cylindrical joint connecting the links i and j is composed of two kinematic elements:

- the mating shaft (body i) or journal;
- the hole (body j) or bearing.

Two Cartesian coordinates frames are attached to these elements.
The k^{th} joint frame on link i is $P_{i_k} - x_{i_k} y_{i_k} z_{i_k}$. The k^{th} joint frame on link j is $P_{j_k} - x_{j_k} y_{j_k} z_{j_k}$.
Origin P_{i_k} is in the middle of the shaft axis. Under ideal working conditions the z_{i_k} and z_{j_k} axes are always collinear. Hence the matrix which transform dual vectors from the first to the second frame is

$$\left[{}^{j_k}_{i_k}\widehat{A}\right] = \begin{bmatrix} \cos\widehat{\theta}_i & -\sin\widehat{\theta}_i & 0 \\ \sin\widehat{\theta}_i & \cos\widehat{\theta}_i & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{6}$$

where $\widehat{\theta}_i = \theta_i + \varepsilon s_i$ is the dual angle formed by line versors $\widehat{i}_{i_k}$ and $\widehat{i}_{j_k}$. Hence, the transform matrix from reference system $O_i - x_i y_i z_i$ on link i to $O_j - x_j y_j z_j$, on link j is

$$\left[{}^{j}_{i}\widehat{A}\right] = \left[{}^{i_k}_{i}\widehat{A}\right] \left[{}^{j_k}_{i_k}\widehat{A}\right] \left[{}^{j}_{j_k}\widehat{A}\right] \tag{7}$$

In the presence of clearance, it is assumed that:

- the kinematic elements have two or an infinite number of points of contact;
- when there are two points of contact, namely B_1 and B_2, these are on the circular edges at the extremities of the shorter element (journal or bearing);
- there is roundness or cylindricity tolerance (radial distance between concentric boundaries) on both shaft and hole;
- the axes of all cross-sectional elements of the shaft (hole) are coincident with the axis of the entire shaft (hole)
- there is not any circular runout;

Due to the existence of radial clearances, the position of the journal axis is deviated from the ideal to an actual position. Such deviation is the *position error*.

Let us introduce a third Cartesian reference frame $P_{i'_k} - x_{i'_k} y_{i'_k} z_{i'_k}$. Under ideal conditions this will coincide with $P_{i_k} - x_{i_k} y_{i_k} z_{i_k}$. Because of clearances, a screw motion is required to superimpose $P_{i_k} - x_{i_k} y_{i_k} z_{i_k}$ on $P_{i'_k} - x_{i'_k} y_{i'_k} z_{i'_k}$.

The axis $z_{i'_k}$ is aligned with z_{j_k}.

Let C_1 and C_2 be the centers of the circular sections at the extremities of the shaft.

Our purpose is:

- to estimate the parameters of this screw motion as a function of the position of the contact points B_1 and B_2;
- to compute the transform matrix $\left[{}^{i'_k}_{i_k}\widehat{A}\right]$ from one reference system to another.

Due to axes misalignment, the transform matrix from reference system $O_i - x_i y_i z_i$ on link i to $O_j - x_j y_j z_j$, on link j is

$$\left[{}^{j}_{i}\widehat{A}\right] = \left[{}^{i_k}_{i}\widehat{A}\right]\left[{}^{i'_k}_{i_k}\widehat{A}\right]\left[{}^{j_k}_{i'_k}\widehat{A}\right]\left[{}^{j}_{j_k}\widehat{A}\right] \tag{8}$$

In Figure 2 it is shown the displaced position of the shaft. The section of the journal bearing in a plane Π orthogonal to the axis of the bearing is depicted in Figure 3. This plane contains $P_{i'_k}$.

The sections of the bearing extremities on such a plane are ellipses. However, as shown in Figure 4, in the present treatment are considered as circles. This approximation is maintained also in the algebraic computations.

The projections of such points on Π are C'_1 and C'_2, respectively.

The coordinates of such points in the Cartesian system of axes $P_{i'_k} x_{i'_k} y_{i'_k} z_{i'_k}$, with reference to the geometry of Figures 2 and 4,

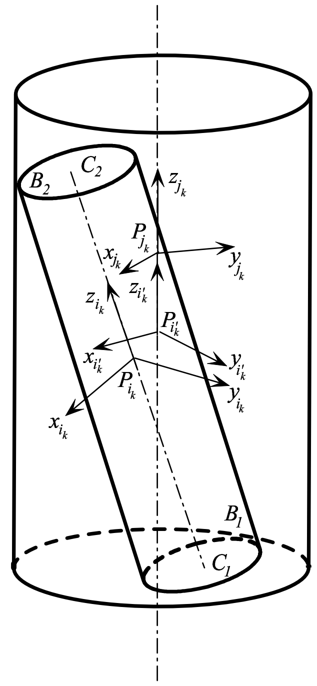

Figure 2. Journal bearing with radial clearance

are[1]

$$x_{C_1} = x_{C'_1} \approx (R-r)\cos\left(\xi_{ij} + \frac{\beta_{ij}}{2}\right),$$
$$y_{C_1} = y_{C'_1} \approx (R-r)\sin\left(\xi_{ij} + \frac{\beta_{ij}}{2}\right),$$
$$z_{C_1} \approx -\frac{H}{2},$$

and

$$x_{C_2} = x_{C'_2} \approx (R-r)\cos\left(\xi_{ij} - \frac{\beta_{ij}}{2}\right),$$
$$y_{C_2} = y_{C'_2} \approx (R-r)\sin\left(\xi_{ij} - \frac{\beta_{ij}}{2}\right),$$
$$z_{C_2} \approx \frac{H}{2},$$

therefore

$$\{C_1C_2\} = \left\{\, x_{C_2} - x_{C_1} \;\; y_{C_2} - y_{C_1} \;\; z_{C_2} - z_{C_1} \,\right\}^T . \tag{9}$$

During Monte Carlo simulation the values of $0 \le \beta_{ij} \le 360°$ and $0 \le \xi_{ij} \le 360°$ are considered as stochastic variables with

[1] In all the expressions the factor $\cos\tau_{ij} \approx 1$ has been omitted for simplicity.

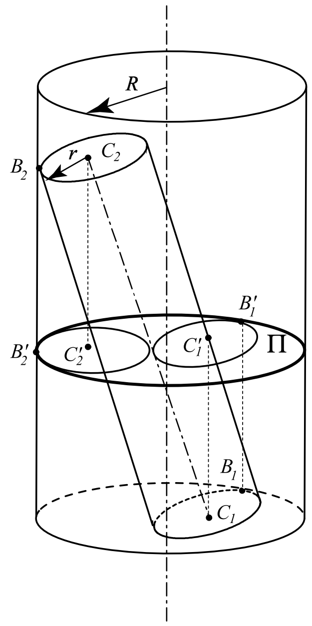

Figure 3. Journal bearing with radial clearance

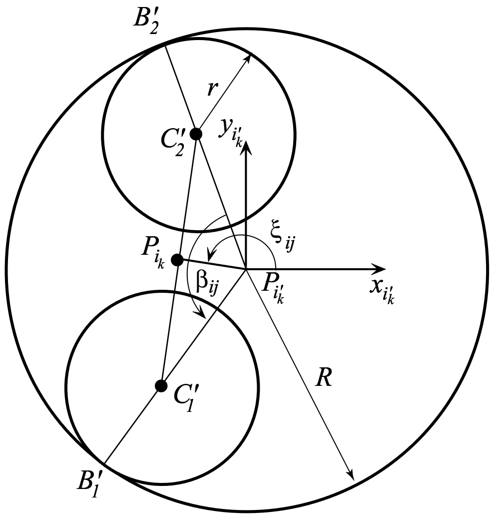

Figure 4. Approximation of journal bearing section

uniform probability density function. All the remaining stochastic parameters follow a Gaussian distribution with standard deviation equal one sixth of the maximum allowed deviation range.

When the contact mode is the one shown in Figure 3, the dual angle $\widehat{\tau}_{ij}$ and the dual unit vector $\widehat{E}_{ij}$ of minimum distance between z_{i_k} and $z_{i'_k}$ can be computed by means of the following procedure:

1. Define the dual vectors

$$\widehat{C_1C_2} = \overrightarrow{C_1C_2} + \varepsilon \overrightarrow{P_{i'_k}C_1} \times \overrightarrow{C_1C_2}\ .$$

$$\widehat{Z}_{i'_k} = \left\{ 0\ 0\ 1 \right\}^T$$

2. Compute the dual unit vector

$$\widehat{E}_{ij} = \frac{\widehat{C_1C_2} \times \widehat{Z}_{i'_k}}{\left\| \widehat{C_1C_2} \times \widehat{Z}_{i'_k} \right\|}$$

3. Compute

$$\cos\widehat{\tau}_{ij} = \frac{\widehat{C_1C_2} \cdot \widehat{Z}_{i'_k}}{\left\| \widehat{C_1C_2} \right\|}$$

$$\sin\widehat{\tau}_{ij} = \left\| \frac{\widehat{C_1C_2}}{\left\| \widehat{C_1C_2} \right\|} \times \widehat{Z}_{i'_k} \right\|$$

4. Compute

$$\widehat{\tau}_{ij} = \operatorname{atan2}\left(\sin\widehat{\tau}_{ij}, \cos\widehat{\tau}_{ij}\right)$$

The transform matrix is

$$\left[{}^{i'_k}_{i_k}\widehat{A}\right] = \begin{bmatrix} \widehat{E}_x^2 v\widehat{\tau}_{ij} + c\widehat{\tau}_{ij} & \widehat{E}_x\widehat{E}_y v\widehat{\tau}_{ij} - \widehat{E}_z s\widehat{\tau}_{ij} & \widehat{E}_x\widehat{E}_z v\widehat{\tau}_{ij} + \widehat{E}_y s\widehat{\tau}_{ij} \\ \widehat{E}_x\widehat{E}_y v\widehat{\tau}_{ij} + \widehat{E}_z s\widehat{\tau}_{ij} & \widehat{E}_y^2 v\widehat{\tau}_{ij} + c\widehat{\tau}_{ij} & \widehat{E}_y\widehat{E}_z v\widehat{\tau}_{ij} - \widehat{E}_y s\widehat{\tau}_{ij} \\ \widehat{E}_x\widehat{E}_z v\widehat{\tau}_{ij} - \widehat{E}_y s\widehat{\tau}_{ij} & \widehat{E}_y\widehat{E}_z v\widehat{\tau}_{ij} + \widehat{E}_x s\widehat{\tau}_i & \widehat{E}_z^2 v\widehat{\tau}_{ij} + c\widehat{\tau}_{ij} \end{bmatrix} \tag{10}$$

where $v\widehat{\tau}_{ij} = 1 - \cos\widehat{\tau}_{ij}$, $c\widehat{\tau}_{ij} = \cos\widehat{\tau}_{ij}$, $s\widehat{\tau}_{ij} = \sin\widehat{\tau}_{ij}$, $\widehat{E}_{ij} = \left\{ \widehat{E}_x\ \widehat{E}_y\ \widehat{E}_z \right\}^T$.

After the finite motion described by this transform the origin P_{i_k} is coincident with $P_{i'_k}$ and the axis z_{i_k} is aligned with $z_{i'_k}$. The axes $x_{i'_k}$ and $y_{i'_k}$ are, respectively, those occupied by the axes x_{i_k} and y_{i_k} right after this finite motion.

The second motion of interest for the full overlap of Cartesian system of axes $P_{i_k} - x_{i_k}y_{i_k}z_{i_k}$ on $P_{j_k} - x_{j_k}y_{j_k}z_{j_k}$, is a screw motion about the axis $z_{i'_k} \equiv z_{j_k}$ described by the transform

$$\left[{}^{j_k}_{i'_k}\widehat{A}\right] = \begin{bmatrix} \cos\widehat{\theta}_i & -\sin\widehat{\theta}_i & 0 \\ \sin\widehat{\theta}_i & \cos\widehat{\theta}_i & 0 \\ 0 & 0 & 1 \end{bmatrix}\ , \tag{11}$$

where the current value of dual angle $\widehat{\theta}_i = \theta_i + \varepsilon s_i$ is computed by solving the loop closure equations[2].
This procedure fails when C_1C_2 and $Z_{i'_k}$ are parallel. In this case the journal and the bearing have a common line of contact, $\widehat{\tau}_{ij} = \varepsilon(R-r)$ and $\widehat{E}_{ij}$ has the same direction of the segment $P_{i_k}P_{i'_k}$.

NUMERICAL EXAMPLE

The procedure has been applied to a Cardan[3] joint herein modelled as an RCCC spatial linkage whose nominal parameters have been summarized in Table 1. For easy comparison, the choice of joint reference frames is such that the meaning of θ_i and s_i is consistent with the one by [8]. Because of the particular kinematic structure of the mechanism, the link fixed reference frame is coincident with one of the joint-fixed reference frame (see Figure 6). Thus, with reference to the geometry of Figure 6, the following equalities hold:

- Kinematic element of joint 1 of link i;

$$P_{i_1} \equiv \left\{\, 0,\, 0,\, 0 \,\right\}^T \;, \tag{12a}$$

$$R_{i_1} \equiv \left\{\, 1,\, 0,\, 0 \,\right\}^T \;, \tag{12b}$$

$$Q_{i_1} \equiv \left\{\, 0,\, 0,\, 1 \,\right\}^T \;, \tag{12c}$$

- Kinematic element of joint 2 of link i;

$$P_{i_2} \equiv \left\{\, a_i,\, R_i \sin\alpha_i,\, R_i(\cos\alpha_i - 1) \,\right\}^T \;, \tag{13a}$$

$$Q_{i_2} \equiv \left\{\, a_i,\, (R_i+1)\sin\alpha_i,\, (R_i+1)\cos\alpha_i - R_i \,\right\}^T \;, \tag{13b}$$

$$R_{i_2} \equiv \left\{\, 1 + a_i,\, R_i \sin\alpha_i,\, R_i(\cos\alpha_i - 1) \,\right\}^T \;, \tag{13c}$$

Clearances are assumed only in cylindrical joints. The revolute joint connecting the driving link and the frame has no clearances. The input angular speed is $\omega_1 = \pi/2$ rad/s. The unknowns of position kinematic analysis are the dual angles $\widehat{\theta}_i = \theta_i + \varepsilon s_i$ $(i = 2,3,4)$. The output angular speed is ω_4. At every mechanism configuration the random number generators, for each of the three cylindrical pairs, compute a set of stochastic variables β_{ij} and ξ_{ij} with uniform probability density function. Similarly, the sets of dimension parameters summarized in Table 1 have been computed assigning a Gaussian distribution with standard deviation equal one sixth of the maximum allowed deviation range.
For each set member execute the kinematic analysis. The computed angles, displacements and velocities are obtained as the average of the values corresponding to a generated set of stochastic variables. In this particular example, each set of a given stochastic variable contains 50 members. It has been observed that an increase of this number does not affect sensitively the results. The Matlab procedures used to generate the random numbers according to the Gaussian and the uniform probability distribution are `normrnd` and `rand`, respectively.

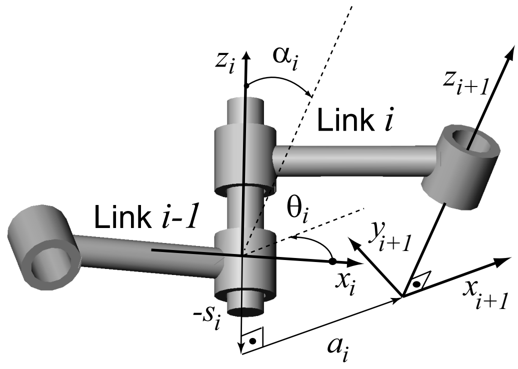

Figure 5. Denavit-Hartenberg parameters

Table 1. Variation range of Denavit-Hartenberg parameters (see Figure 5) and other stochastic variables.

Link	α_i	a_i	R_i
1	$90° \pm 0.1°$	0 ± 0.1	1 ± 0.1
2	$90° \pm 0.1°$	0 ± 0.1	1 ± 0.1
3	$90° \pm 0.1°$	0 ± 0.1	1 ± 0.1
4	$135° \pm 0.1°$	0 ± 0.1	1 ± 0.1
H	Δr	ξ	β
1 ± 0.1	0 ± 0.06	0°-360°	0°-360°

[2] In the case of the RCCC spatial linkage considered in this paper equation (5) express the loop closure condition.

[3] For a correct interpretation of all the plots presented in this section, it should be acknowledged that the ideal kinematic structure of a Cardan joint is a spherical four-bar RRRR linkage with all the moving links having α_i angles equal to 90 degrees. However, the *RRRR* is an overconstrained mechanism whose mobility depends on the fulfillment of geometric conditions, such as the convergence of all revolute axes in the same point. For this reason, in technical literature e.g. [13, 14], the effects of manufacturing errors on the kinematics and dynamics of this particular joint are usually monitored through the analysis of an RCCC linkage whose mobility does not depend on the previously mentioned condition. Thus one can study the influence of the missed intersection of kinematic pairs axes or of link dimensional variations on the kinematics of a Cardan joint through the analysis of an RCCC linkage with prescribed axes misalignments.

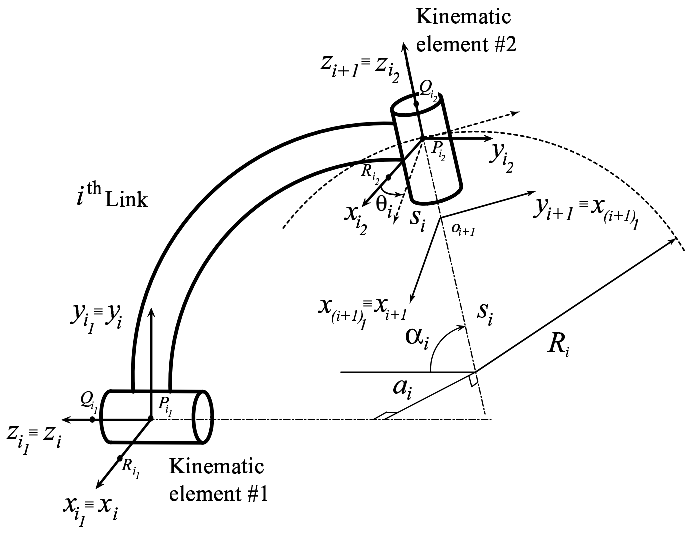

Figure 6. Positions of the i-th link fixed reference frame and of the two joint reference frames in a generic link of the RCCC mechanism.

The continuous plots represent the ideal case without errors or tolerances (the mechanism is an RRRR linkage with all revolute axes converging in one point only). The dotted plots represent the average values of the output angles, displacements and velocities.

CONCLUSIONS

It has been discussed the use of dual numbers for the statistical modeling of manufacturing tolerances in spatial linkages. Monte Carlo type of simulation has been used for the kinematic analysis. Some stochastic variables required for the kinematic analysis have been randomly generated assuming an uniform probability distribution. The method can handle spatial linkages with cylindrical joints with clearances and links with dimensional tolerances. Geometric tolerances due to axes misalignments can be modeled as well.
The method has been applied to the case of a Cardan joint. For this particular example it seems that:

- Output angles are not very much affected by the dimensional tolerances and clearances in the joints (see Figure 7).
- Displacements and speeds of links along the axes of cylindrical joints are very sensitive to all type of errors (see Figures 9 and 10).
- The angular speed of the output link is sensitive to errors (see Figure 8).
- There are two positions where the standard deviations of all the output kinematic variables reach their maximum value. Likely these are those where the Jacobian matrix of the constraint equations is close to singularity.

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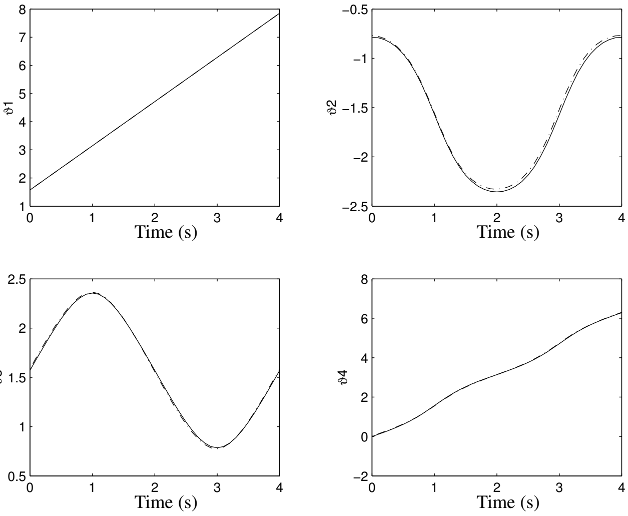

Figure 7. Rotation angles at the joints.

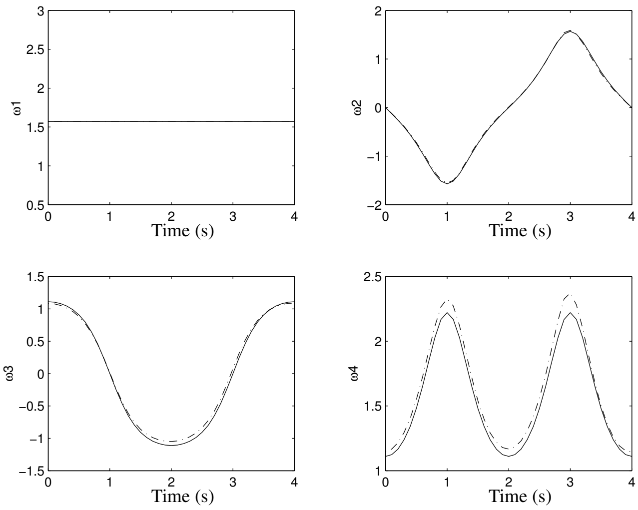

Figure 8. Angular speeds at the joints.

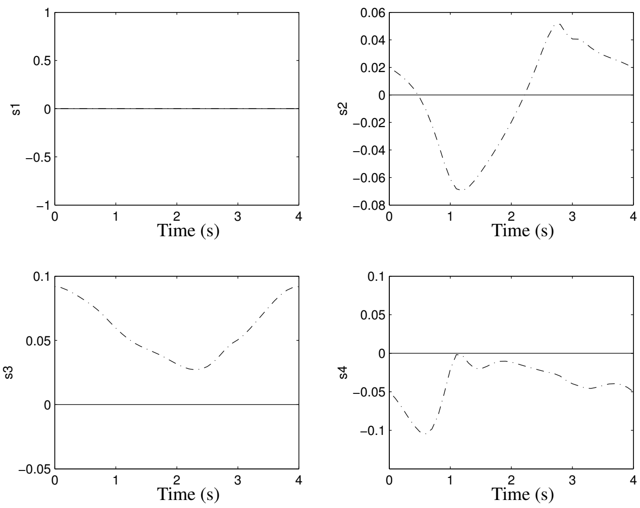

Figure 9. Displacements along the axes of cylindrical joints.

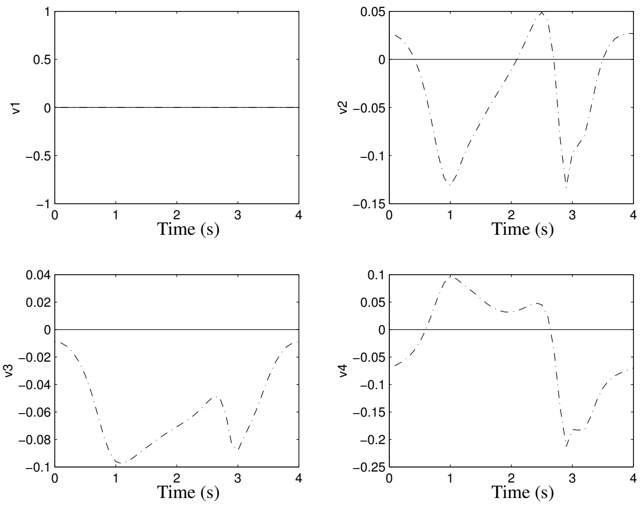

Figure 10. Sliding speeds along the axes of cylindrical joints.

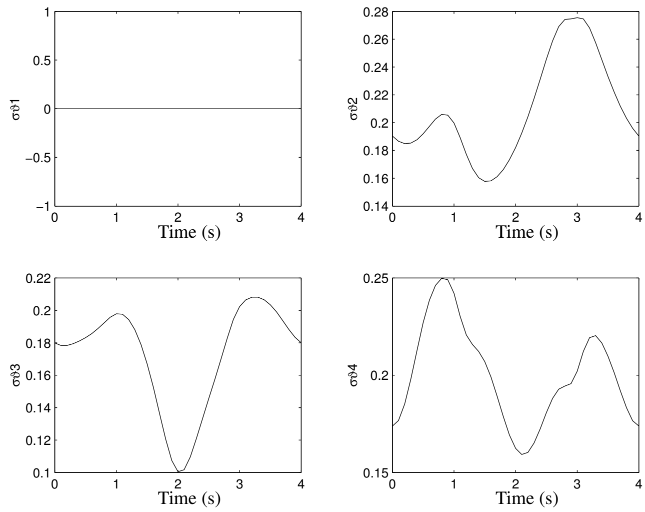

Figure 11. Standard deviation of output angles.

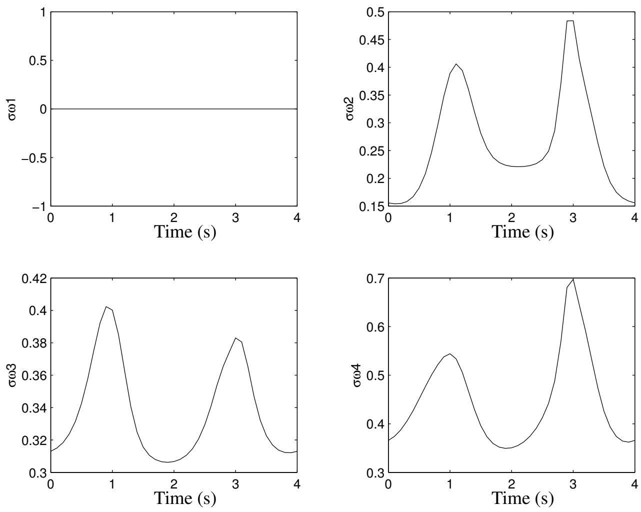

Figure 13. Standard deviation of angular speeds at the joints.

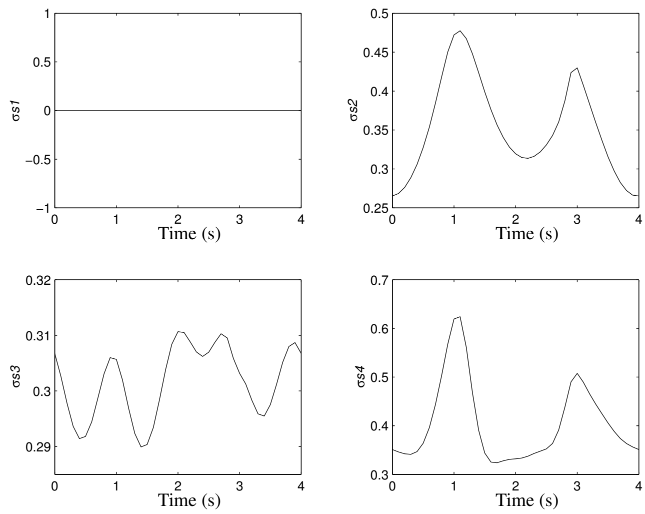

Figure 12. Standard deviation of displacements along the axes of cylindrical joints.

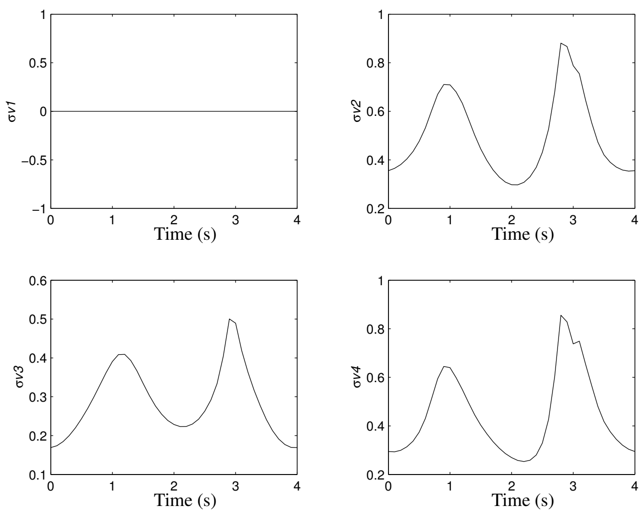

Figure 14. Standard deviation of sliding speeds along the axes of cylindrical joints.